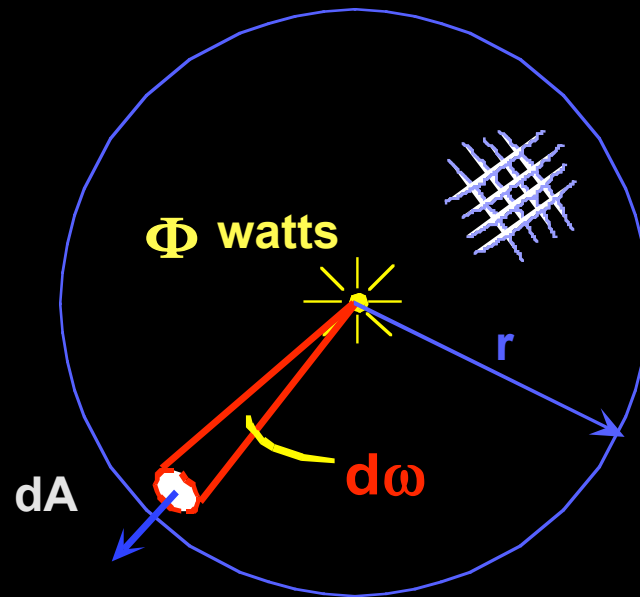


- Relationship between radiance (radiant intensity) and irradiance



$$d\omega = \frac{dA}{r^2}$$

$$E = \frac{d\Phi}{dA}$$

R: Radiant Intensity

E: Irradiance

Φ: Watts

ω : Steradians

$$R = \frac{d\Phi}{d\omega} = \frac{r^2 d\Phi}{dA} = r^2 E$$

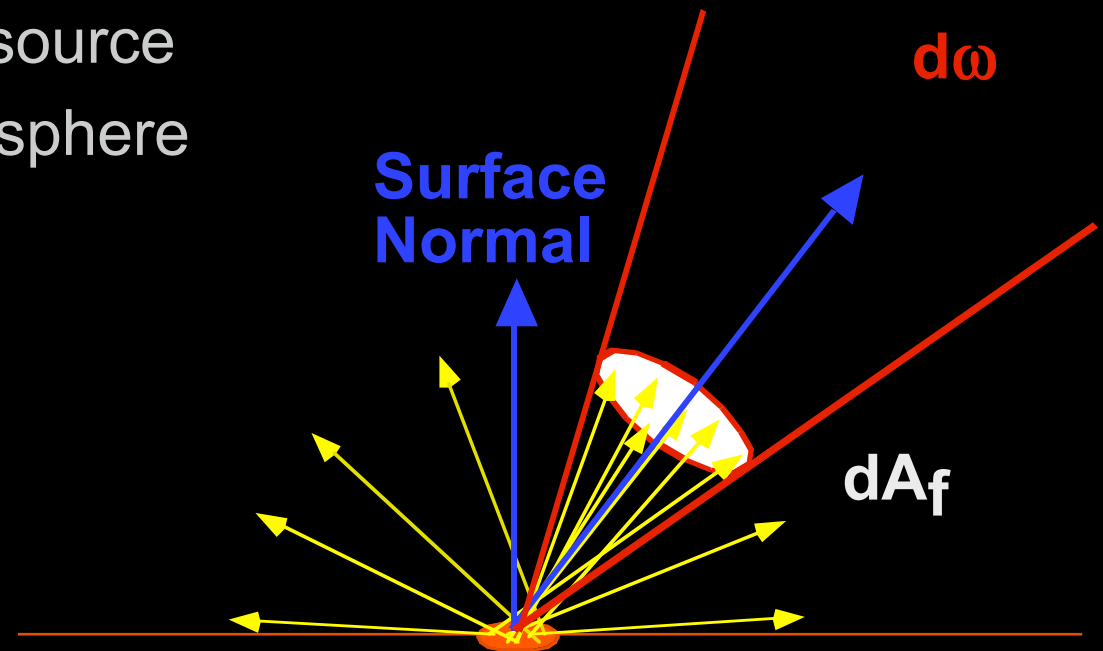
$$E = \frac{R}{r^2}$$

- Surface acts as light source
- Radiates over a hemisphere

R: Radiant Intensity

E: Irradiance

L: Surface radiance



- Surface Radiance: power per unit foreshortened area emitted into a solid angle

$$L = \frac{d^2\Phi}{dA_f d\omega}$$

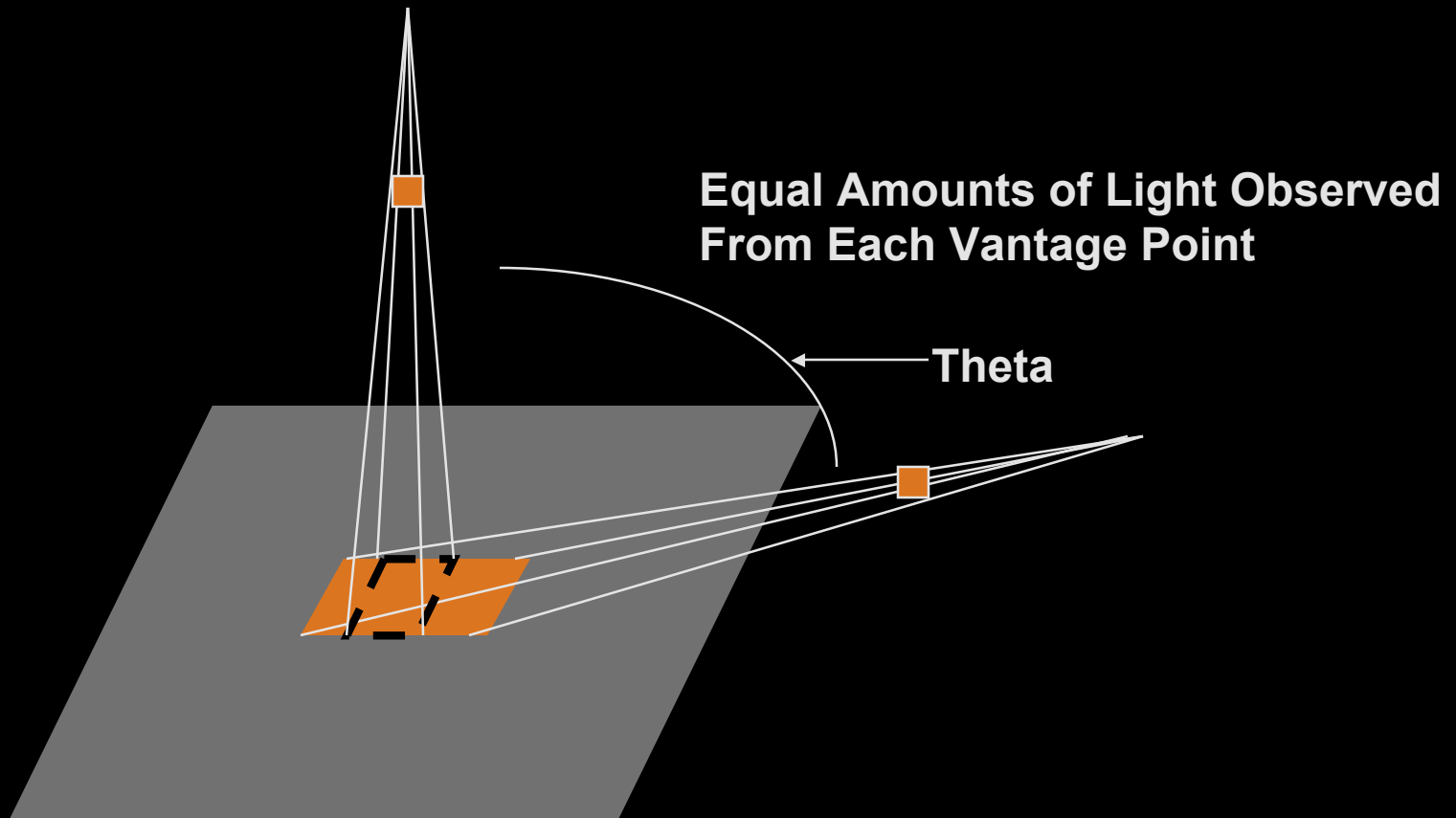
(watts/m² - steradian)

- Consider two definitions:
 - Radiance:
power per unit foreshortened area emitted into a solid angle
 - Pseudo-radiance
power per unit area emitted into a solid angle

- Why should we work with radiance rather than pseudo-radiance?
 - Only reason: Radiance is more closely related to our intuitive notion of “brightness”.

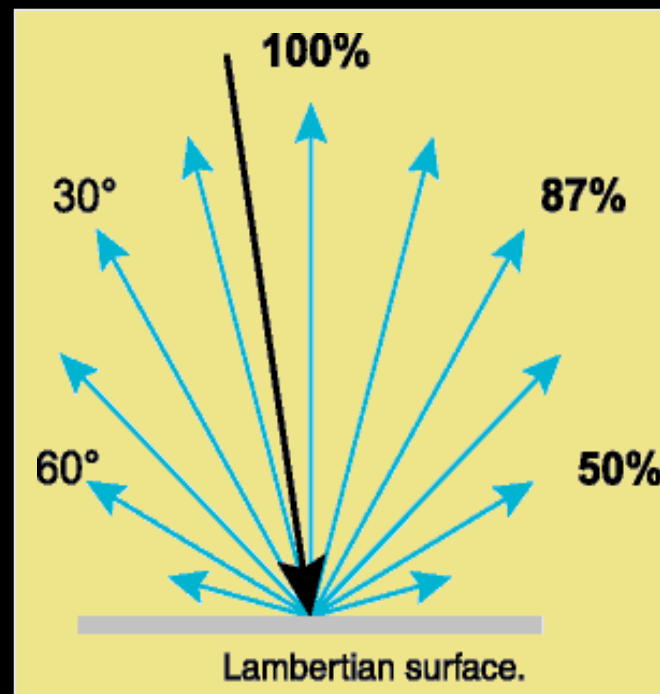


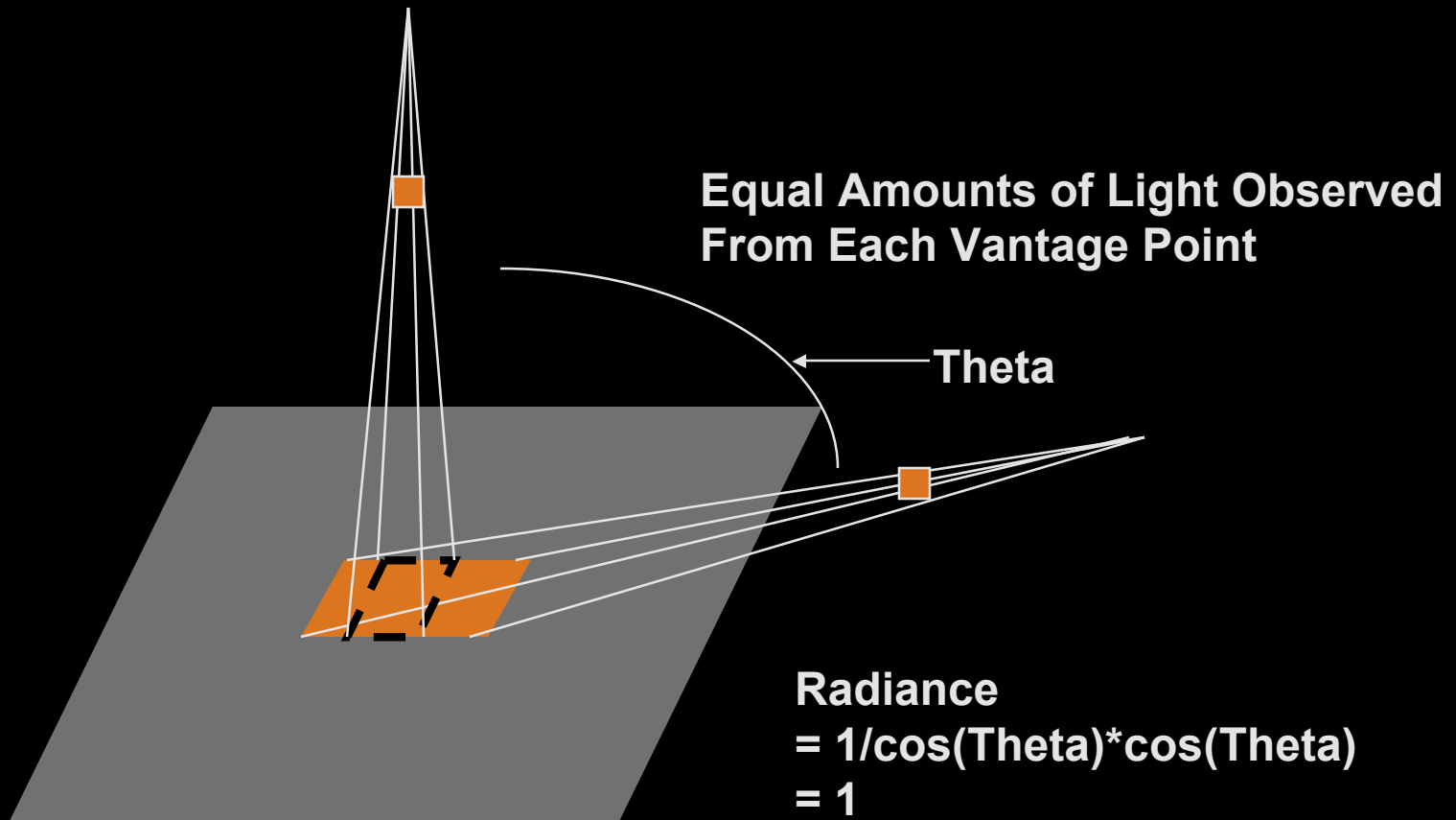
- A particular point P on a Lambertian (perfectly matte) surface appears to have the same brightness no matter what angle it is viewed from.
 - Piece of paper
 - Matte paint
- Doesn't depend upon incident light angle.
- What does this say about how they emit light?



Area of black box = 1
Area of orange box = $1/\cos(\text{Theta})$
Foreshortening rule.

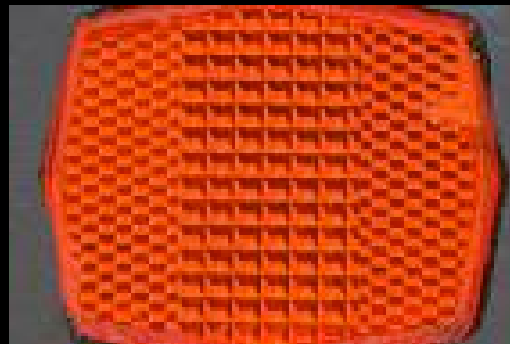
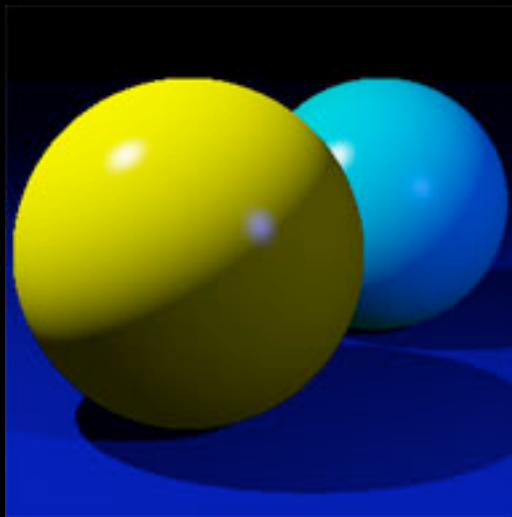
Relative magnitude of light scattered in each direction.
Proportional to $\cos(\theta)$.



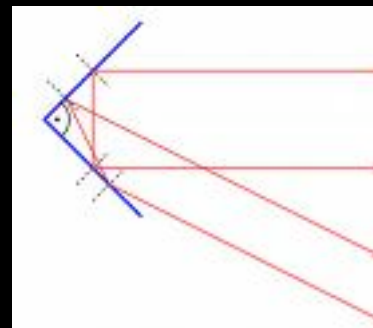


Area of black box = 1
Area of orange box = $1/\cos(\text{Theta})$
Foreshortening rule.

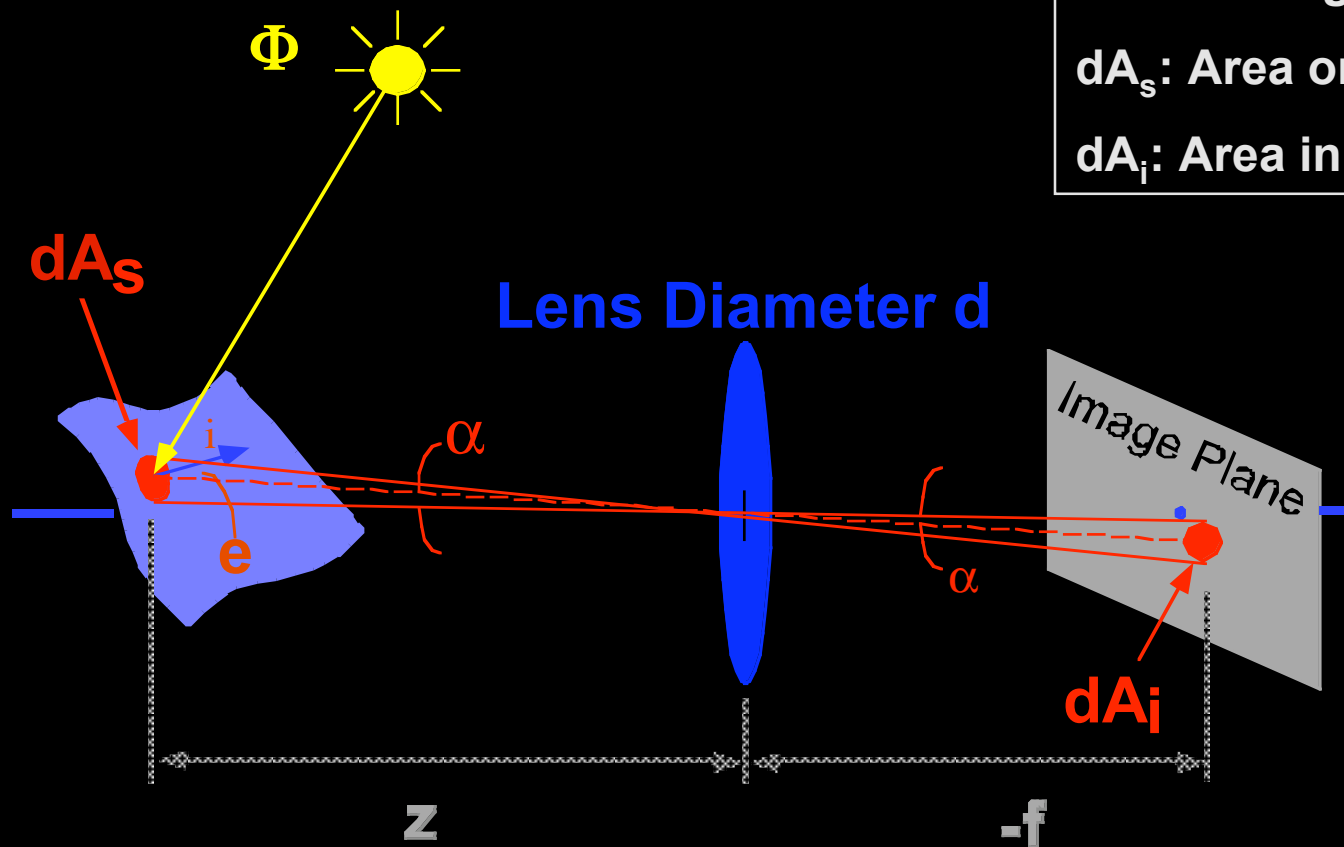
The bidirectional reflectance distribution function.



SWISSPEARL CARAT SL		
White 000	0000	0000
White 001	0001	0001
White 002	0002	0002
White 003	0003	0003
White 004	0004	0004
White 005	0005	0005
White 006	0006	0006
White 007	0007	0007
White 008	0008	0008
White 009	0009	0009
White 010	0010	0010
White 011	0011	0011
White 012	0012	0012
White 013	0013	0013
White 014	0014	0014
White 015	0015	0015
White 016	0016	0016
White 017	0017	0017
White 018	0018	0018
White 019	0019	0019
White 020	0020	0020
White 021	0021	0021
White 022	0022	0022
White 023	0023	0023
White 024	0024	0024
White 025	0025	0025
White 026	0026	0026
White 027	0027	0027
White 028	0028	0028
White 029	0029	0029
White 030	0030	0030
White 031	0031	0031
White 032	0032	0032
White 033	0033	0033
White 034	0034	0034
White 035	0035	0035
White 036	0036	0036
White 037	0037	0037
White 038	0038	0038
White 039	0039	0039
White 040	0040	0040
White 041	0041	0041
White 042	0042	0042
White 043	0043	0043
White 044	0044	0044
White 045	0045	0045
White 046	0046	0046
White 047	0047	0047
White 048	0048	0048
White 049	0049	0049
White 050	0050	0050
White 051	0051	0051
White 052	0052	0052
White 053	0053	0053
White 054	0054	0054
White 055	0055	0055
White 056	0056	0056
White 057	0057	0057
White 058	0058	0058
White 059	0059	0059
White 060	0060	0060
White 061	0061	0061
White 062	0062	0062
White 063	0063	0063
White 064	0064	0064
White 065	0065	0065
White 066	0066	0066
White 067	0067	0067
White 068	0068	0068
White 069	0069	0069
White 070	0070	0070
White 071	0071	0071
White 072	0072	0072
White 073	0073	0073
White 074	0074	0074
White 075	0075	0075
White 076	0076	0076
White 077	0077	0077
White 078	0078	0078
White 079	0079	0079
White 080	0080	0080
White 081	0081	0081
White 082	0082	0082
White 083	0083	0083
White 084	0084	0084
White 085	0085	0085
White 086	0086	0086
White 087	0087	0087
White 088	0088	0088
White 089	0089	0089
White 090	0090	0090
White 091	0091	0091
White 092	0092	0092
White 093	0093	0093
White 094	0094	0094
White 095	0095	0095
White 096	0096	0096
White 097	0097	0097
White 098	0098	0098
White 099	0099	0099
White 100	0100	0100



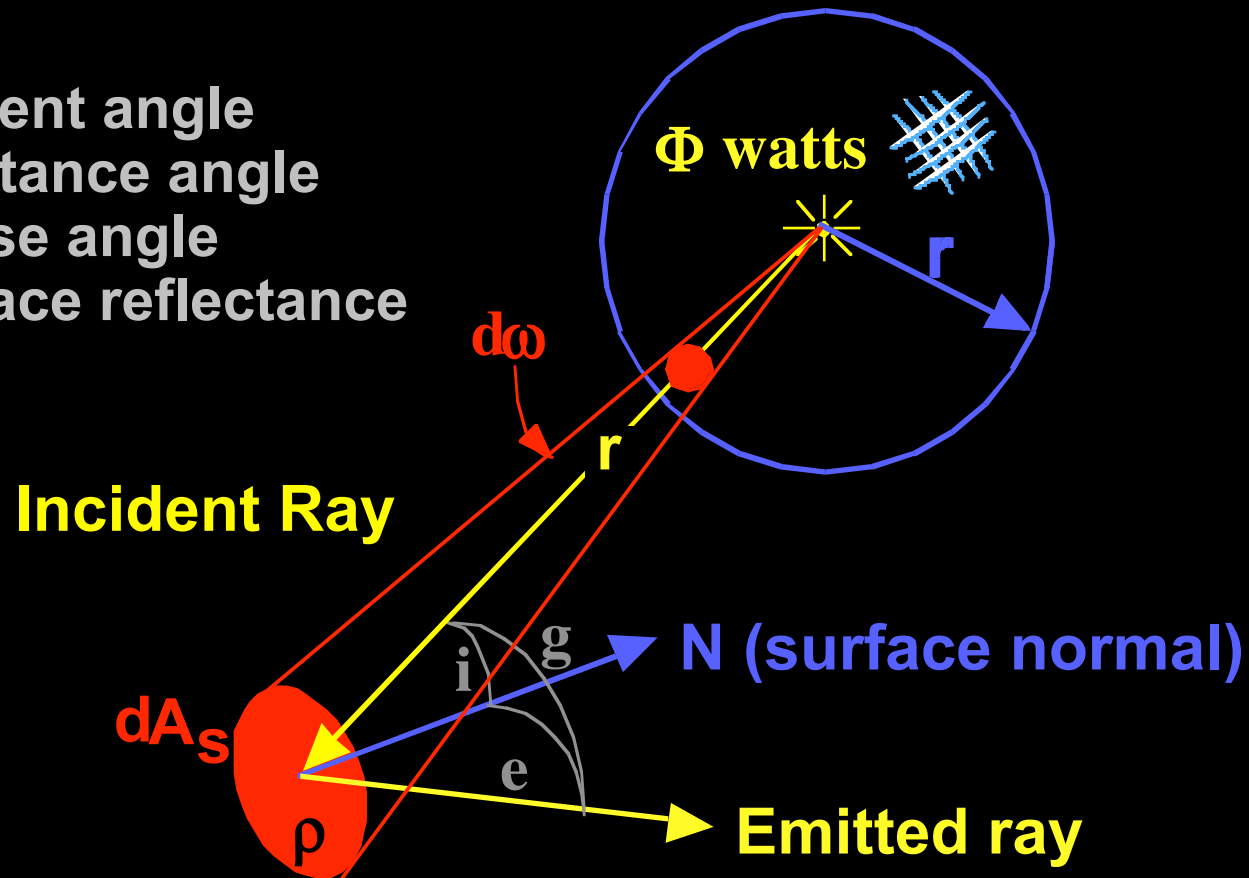
- Goal: Relate the radiance of a surface to the irradiance in the image plane of a simple optical system.



α : Solid angle of patch
 dA_s : Area on surface
 dA_i : Area in image

- $E = \text{flux incident on the surface (irradiance)} = \frac{d\Phi}{dA}$

i = incident angle
 e = emittance angle
 g = phase angle
 ρ = surface reflectance



- We need to determine $d\Phi$ and dA

dA

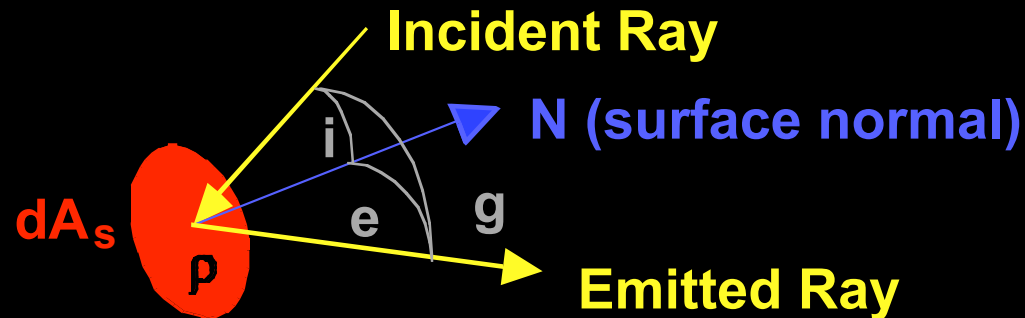
- $dA = dA_s \cos i$ {foreshortening effect in direction of light source}

$d\Phi$

- $d\Phi =$ flux intercepted by surface over area dA
 - dA subtends solid angle $d\omega = dA_s \cos i / r^2$
 - $d\Phi = R d\omega = R dA_s \cos i / r^2$
 - $E = d\Phi / dA_s$

Surface Irradiance: $E = R \cos i / r^2$

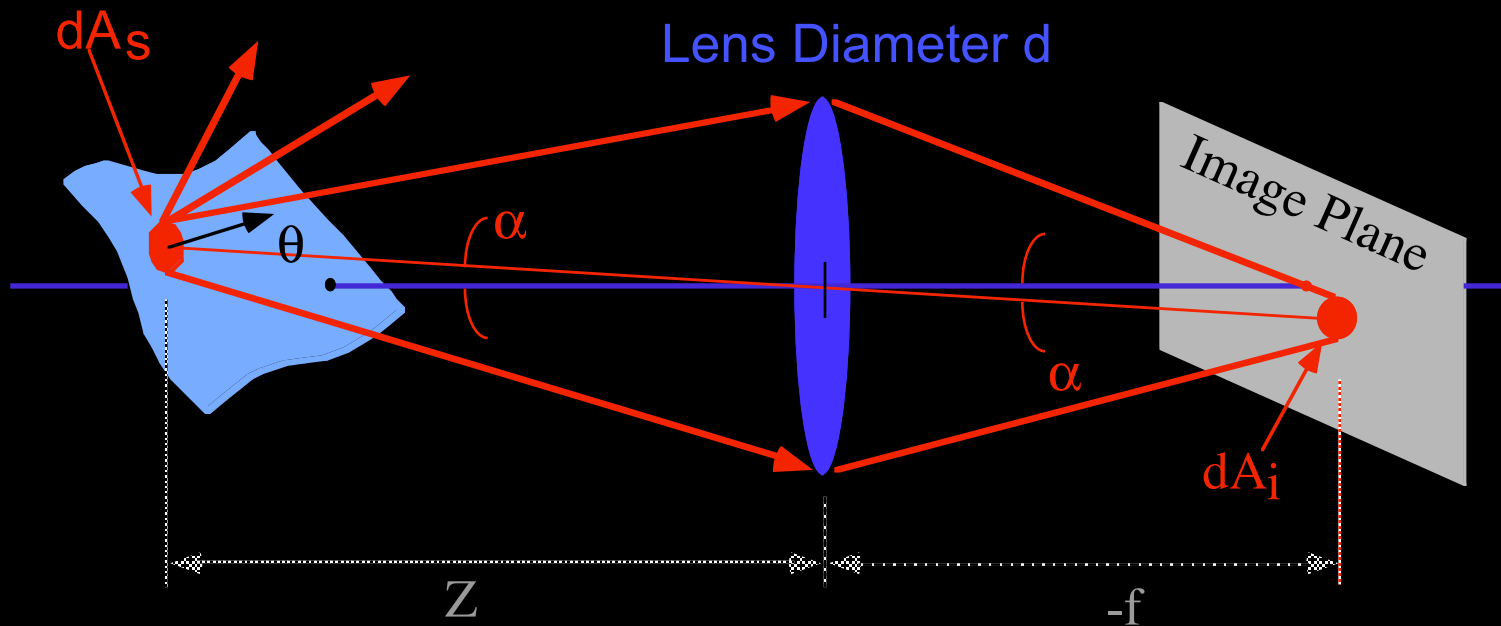
- Now treat small surface area as an emitter
 -because it is bouncing light into the world
- How much light gets reflected?



- E is the surface irradiance
- L is the surface radiance = luminance
- They are related through the surface reflectance function:

$$\frac{L_s}{E} = \rho(i, e, g, \lambda)$$

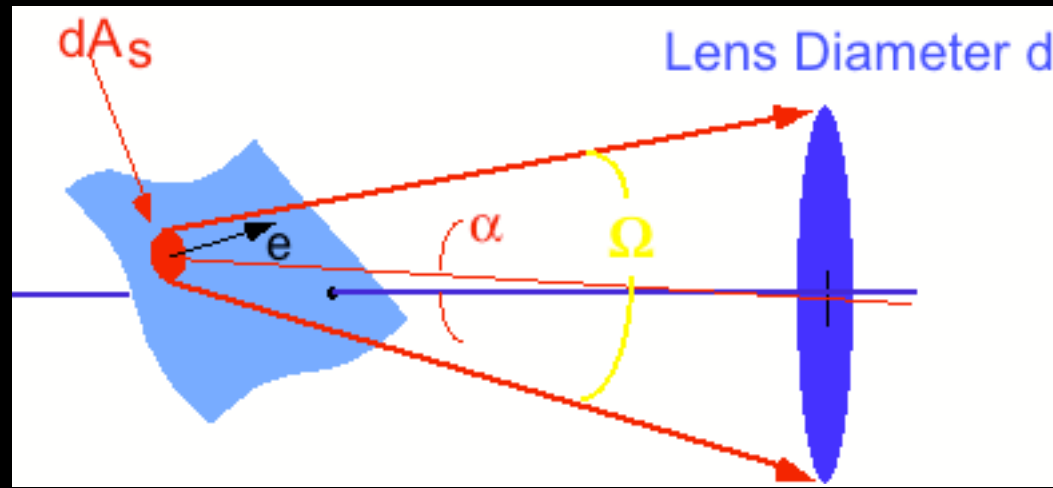
May also be a function of the wavelength of the light



$$L_s = \frac{d^2\Phi}{dA_s d\omega} \quad \text{Luminance of patch (known from previous step)}$$

What is the power of the surface patch as a source in the direction of the lens?

$$d^2\Phi = L_s dA_s d\omega$$



■ In general:

- L_s is a function of the angles i and e .
- Lens can be quite large
- Hence, must integrate over the lens solid angle to get $d\Phi$

$$d\Phi = dA_s \int_{\Omega} L_s d\Omega$$

- Lens diameter is small relative to distance from patch

$$d\Phi = dA_s \int_{\Omega} L_s d\Omega$$

L_s is a constant and can be removed from the integral

$$d\Phi = dA_s L_s \int_{\Omega} d\Omega$$

Surface area of patch in direction of lens

$$= dA_s \cos e$$

Solid angle subtended by lens in direction of patch

$$= \frac{\text{Area of lens as seen from patch}}{(\text{Distance from lens to patch})^2}$$

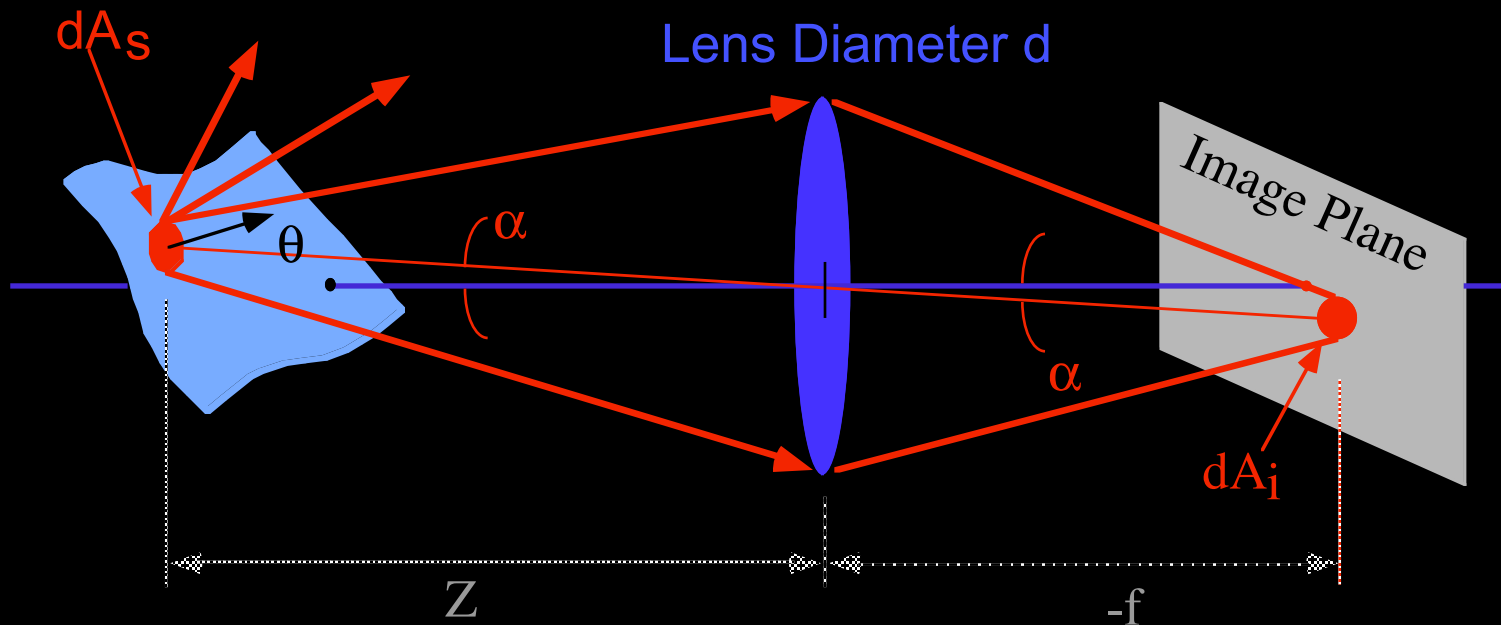
$$= \frac{\pi (d/2)^2 \cos \alpha}{(z / \cos \alpha)^2}$$

$$\begin{aligned}
 d\Phi &= dA_s \int_{\Omega} L_s d\Omega \\
 &= dA_s \cos e L_s \frac{\pi (d/2)^2 \cos \alpha}{(z / \cos \alpha)^2}
 \end{aligned}$$

- Power concentrated in lens:

$$d\Phi = \frac{\pi}{4} L_s dA_s \left[\frac{d}{z} \right]^2 \cos e \cos^3 \alpha$$

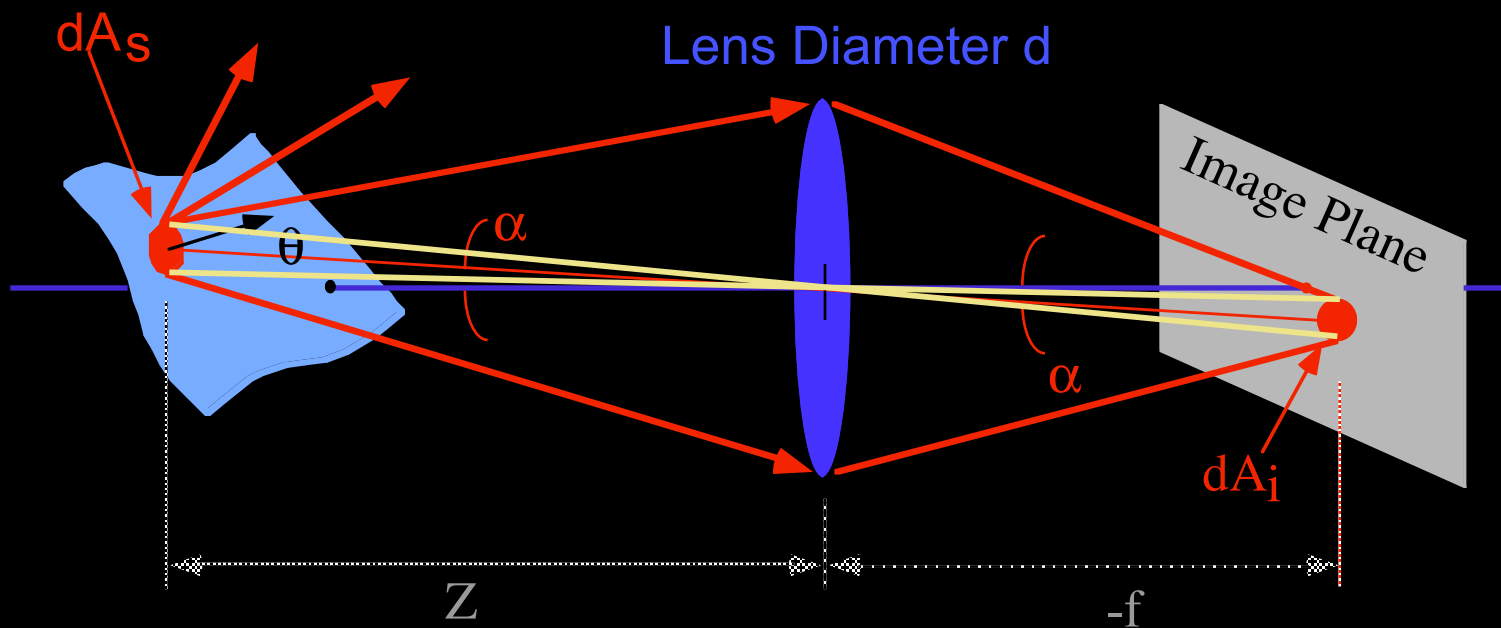
- Assuming a lossless lens, this is also the power radiated by the lens as a source.



- Image irradiance at $dA_i = \frac{d\Phi}{dA_i} = E_i$

$$E_i = L_s \frac{dA_s}{dA_i} \frac{\pi}{4} \left[\frac{d}{z} \right]^2 \cos e \cos^3 \alpha$$

ratio of areas



The two solid angles are equal

$$\frac{dA_s \cos e}{(Z / \cos \alpha)^2} = \frac{dA_i \cos \alpha}{(-f / \cos \alpha)^2}$$



$$\frac{dA_s}{dA_i} = \frac{\cos \alpha}{\cos e} \left(\frac{Z}{-f} \right)^2$$

- Source Radiance to Image Sensor Irradiance:

$$\frac{dA_s}{dA_i} = \frac{\cos \alpha}{\cos e} \left(\frac{Z}{-f} \right)^2$$

$$E_i = L_s \frac{dA_s}{dA_i} \frac{\pi}{4} \left(\frac{d}{Z} \right)^2 \cos e \cos^3 \alpha$$

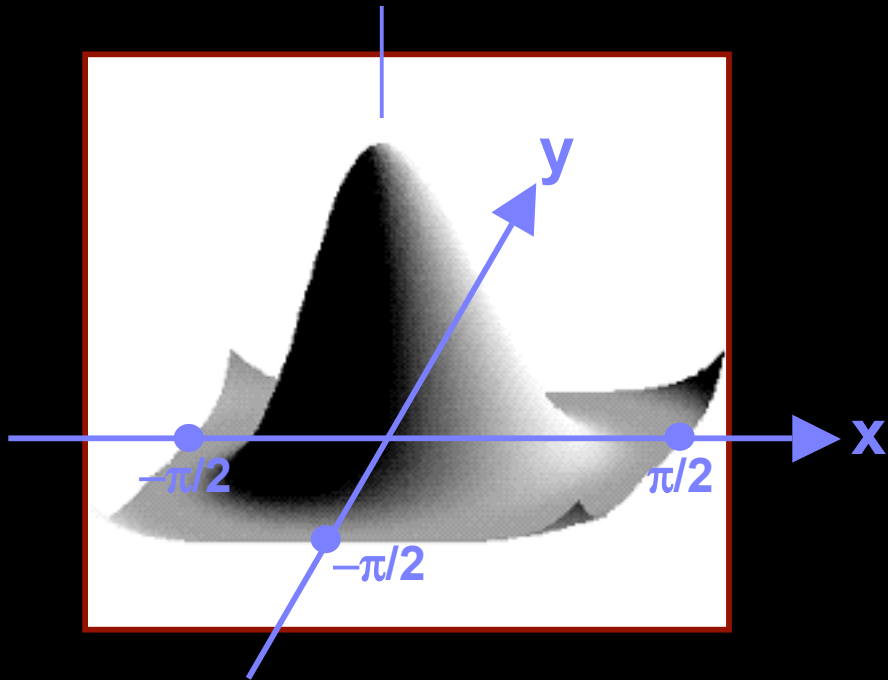
$$E_i = L_s \frac{\cos \alpha}{\cos e} \left(\frac{Z}{-f} \right)^2 \frac{\pi}{4} \left(\frac{d}{Z} \right)^2 \cos e \cos^3 \alpha$$

$$E_i = L_s \frac{\pi}{4} \left(\frac{d}{-f} \right)^2 \cos^4 \alpha$$

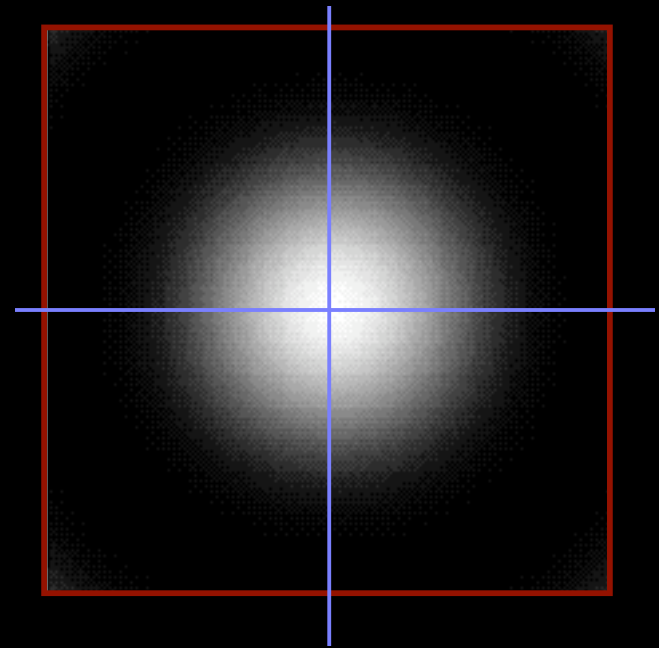
$$E_i = L_s \frac{\pi}{4} \left[\frac{d}{-f} \right]^2 \cos^4 \alpha$$

- Image irradiance is a function of:
 - Scene radiance L_s
 - Focal length of lens f
 - Diameter of lens d
 - f/d is often called the 'effective focal length' of the lens
 - Off-axis angle α

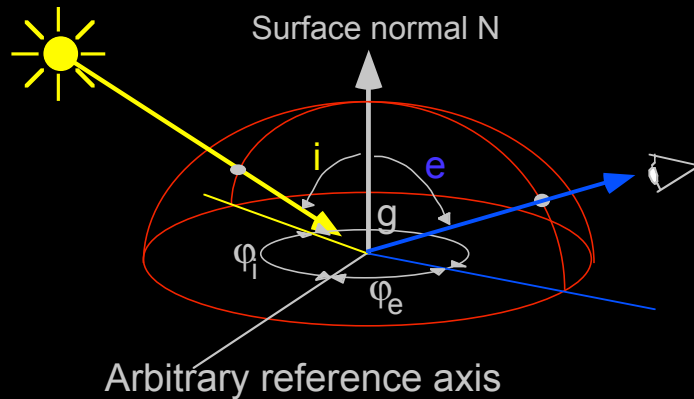
Lens Center



Top view shaded by height



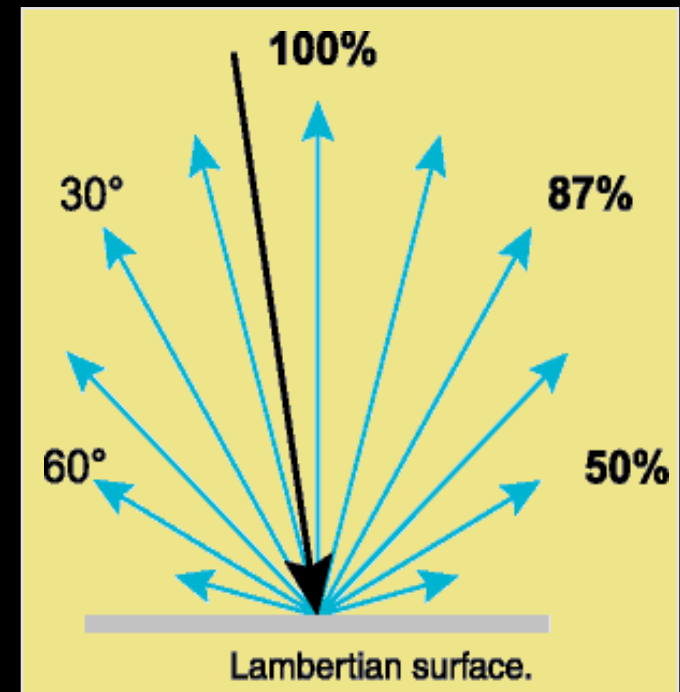
- Surface reflection ρ can be a function of viewing and/or illumination angle



$$\rho(i, e, g, \varphi_e, \varphi_i) = \frac{dL(e, \varphi_e)}{dE(i, \varphi_i)}$$

- ρ may also be a function of the wavelength of the light source
- Assumed a point source (sky, for example, is not)

- The BRDF for a Lambertian surface is a constant
 - $\rho(i, e, g, \phi_e, \phi_r) = k$
 - function of $\cos e$ due to the foreshortening effect
 - k is the 'albedo' of the surface
 - Good model for diffuse surfaces
- Other models combine diffuse and specular components (Phong, Torrance-Sparrow, Oren-Nayar)



■ ■ Introduction to

■ ■ Computer Vision

BRDF

- Ron Dror's thesis

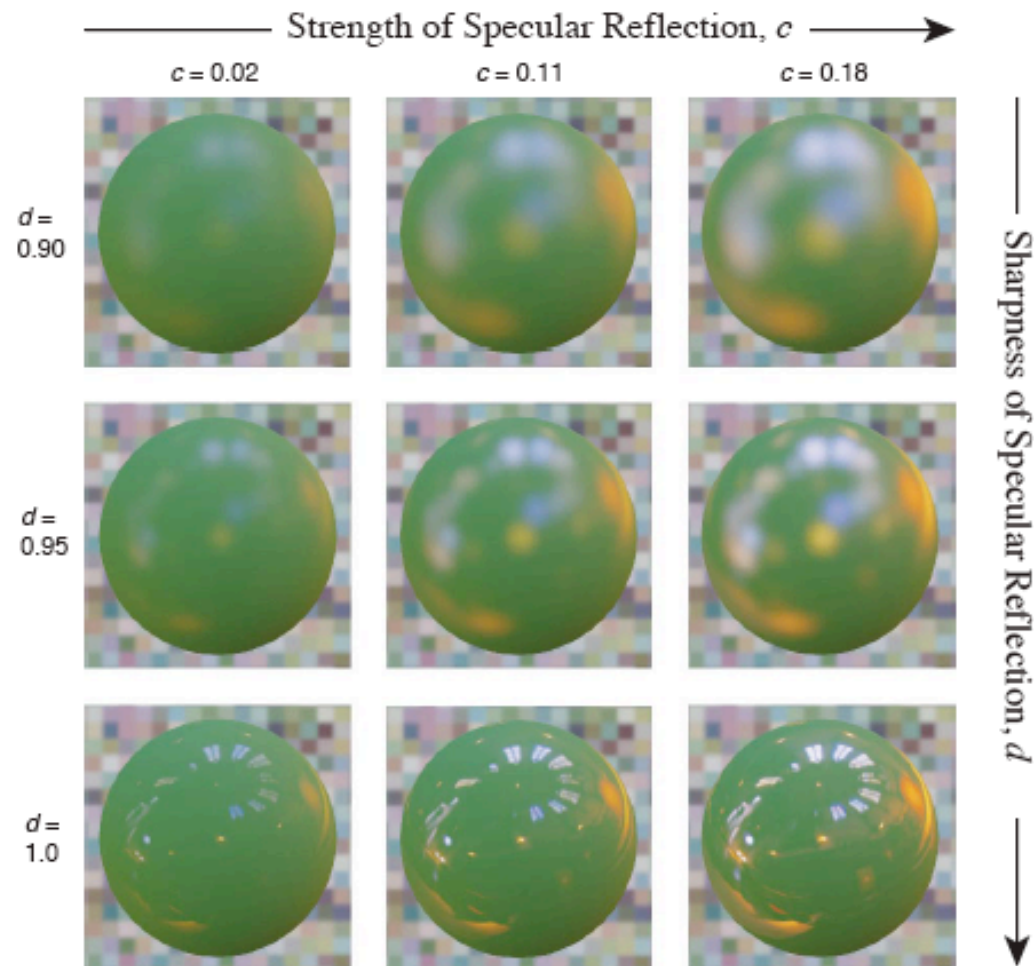
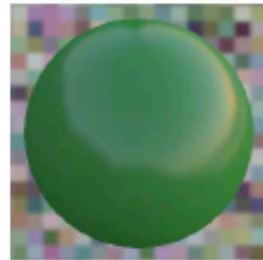


Figure 2.6. Grid showing range of reflectance properties used in the experiments for a particular real-world illumination map. All the spheres shown have an identical diffuse component. In Pellacini's reparameterization of the Ward model, the specular component depends on the c and d parameters. The strength of specular reflection, c , increases with ρ_s , while the sharpness of specular reflection, d , decreases with α . The images were rendered in *Radiance*, using the techniques described in Appendix B.

Real World Illuminations



(a) "Beach"



(b) "Building"



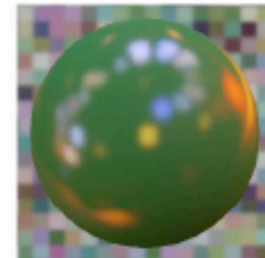
(c) "Campus"



(d) "Eucalyptus"



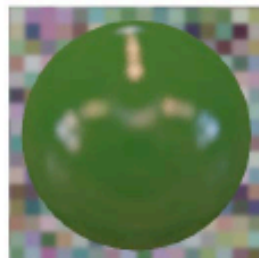
(e) "Galileo"



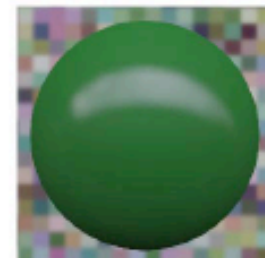
(f) "Grace"



(g) "Kitchen"

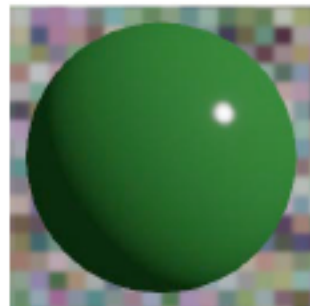


(h) "St. Peter's"

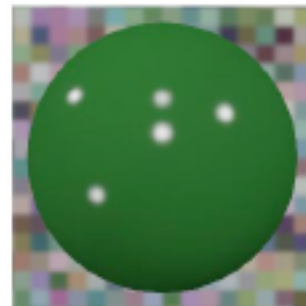


(i) "Uffizi"

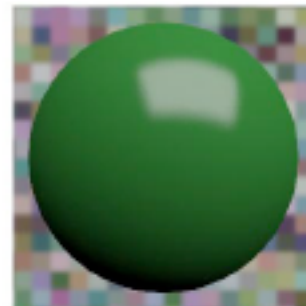
Artificial Illuminations



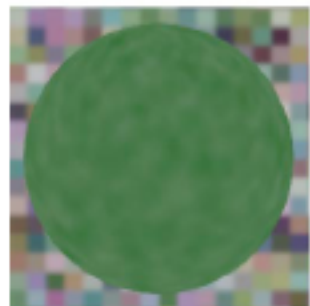
(a) Point source



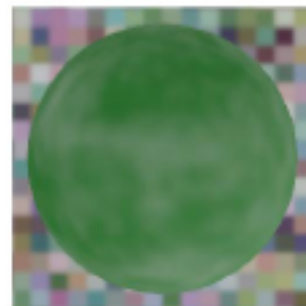
(b) Multiple points



(c) Extended



(d) White noise



(e) Pink noise



(a)



(b)

Figure 2.9. (a) A shiny sphere rendered under illumination by a point light source. (b) The same sphere rendered under photographically-acquired real-world illumination. Humans perceive reflectance properties more accurately in (b).

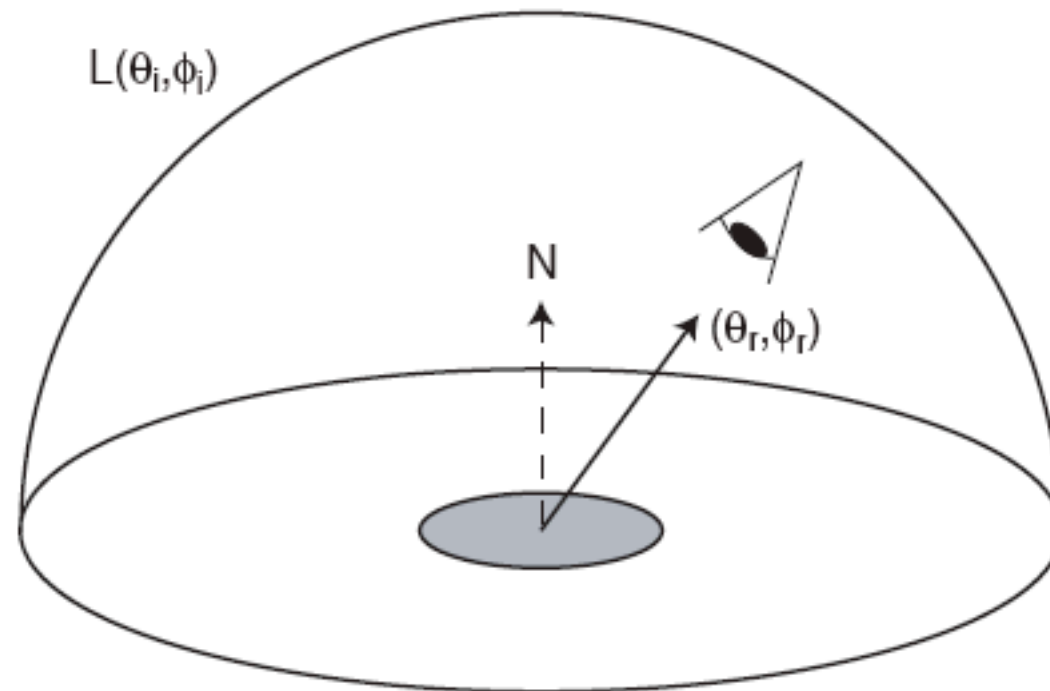


Figure 3.1. A viewer observes a surface patch with normal N from direction (θ_r, ϕ_r) . $L(\theta_i, \phi_i)$ represents radiance of illumination from direction (θ_i, ϕ_i) . The coordinate system is such that N points in direction $(0, 0)$.

$$B(\theta_r, \phi_r) = \int_{\phi_i=0}^{2\pi} \int_{\theta_i=0}^{\pi/2} L(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \sin \theta_i d\theta_i d\phi_i,$$

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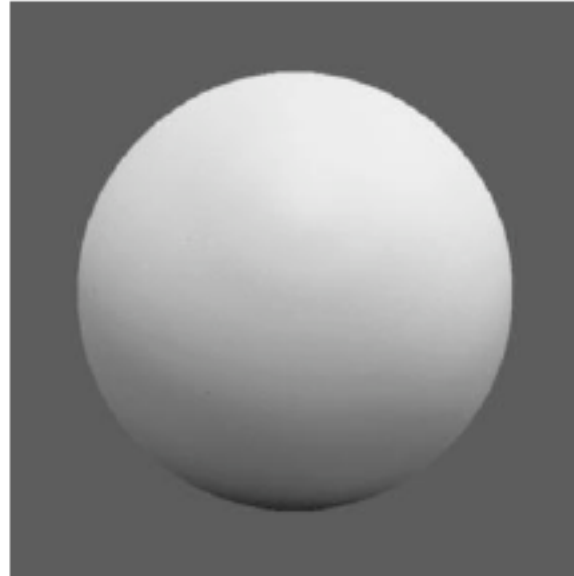


Figure 3.6. A photograph of a matte sphere, shown against a uniform gray background. This image could also be produced by a chrome sphere under appropriate illumination, but that scenario is highly unlikely.