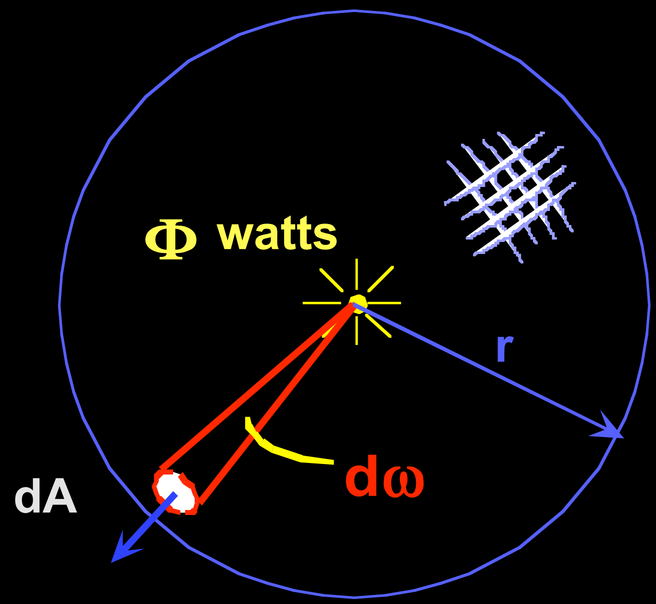
Inverse Square Law

- Relationship between radiance (radiant intensity) and irradiance



$$d\omega = \frac{dA}{r^2}$$

$$E = \frac{d\Phi}{dA}$$

R: Radiant Intensity

E: Irradiance

Φ: Watts

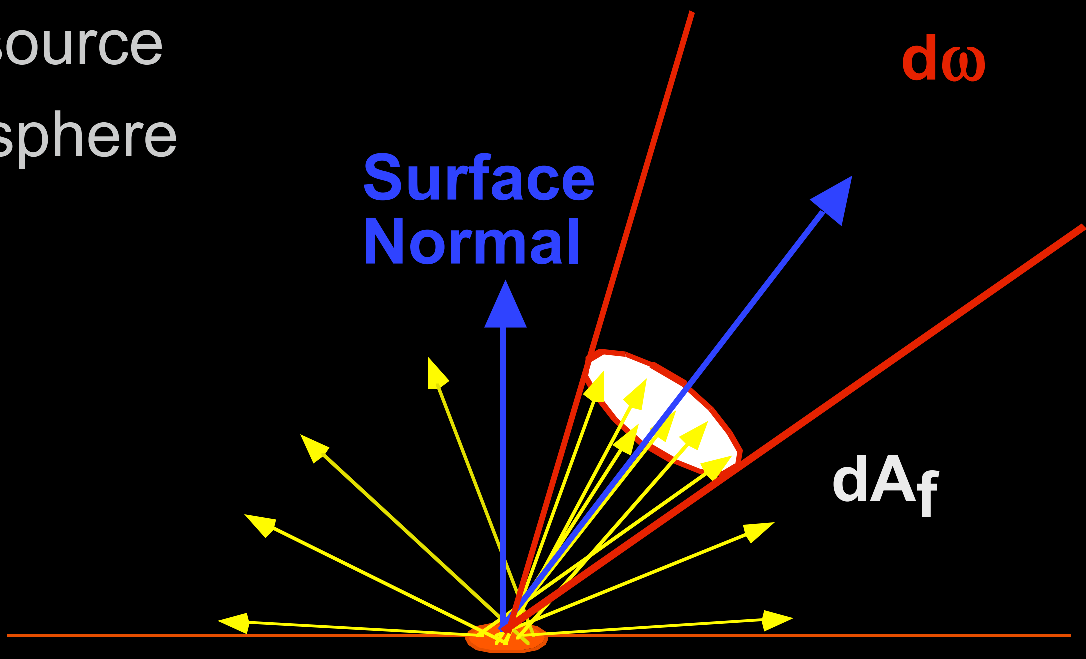
ω : Steradians

$$R = \frac{d\Phi}{d\omega} = \frac{r^2\, d\Phi}{dA} = r^2 E$$

$$E = \frac{R}{r^2}$$

Surface Radiance

- Surface acts as light source
- Radiates over a hemisphere



R: Radiant Intensity
E: Irradiance
L: Surface radiance

- Surface Radiance: power per unit foreshortened area emitted into a solid angle

$$L = \frac{d^2\Phi}{dA_f\, d\omega}$$

(watts/m2 - steradian)

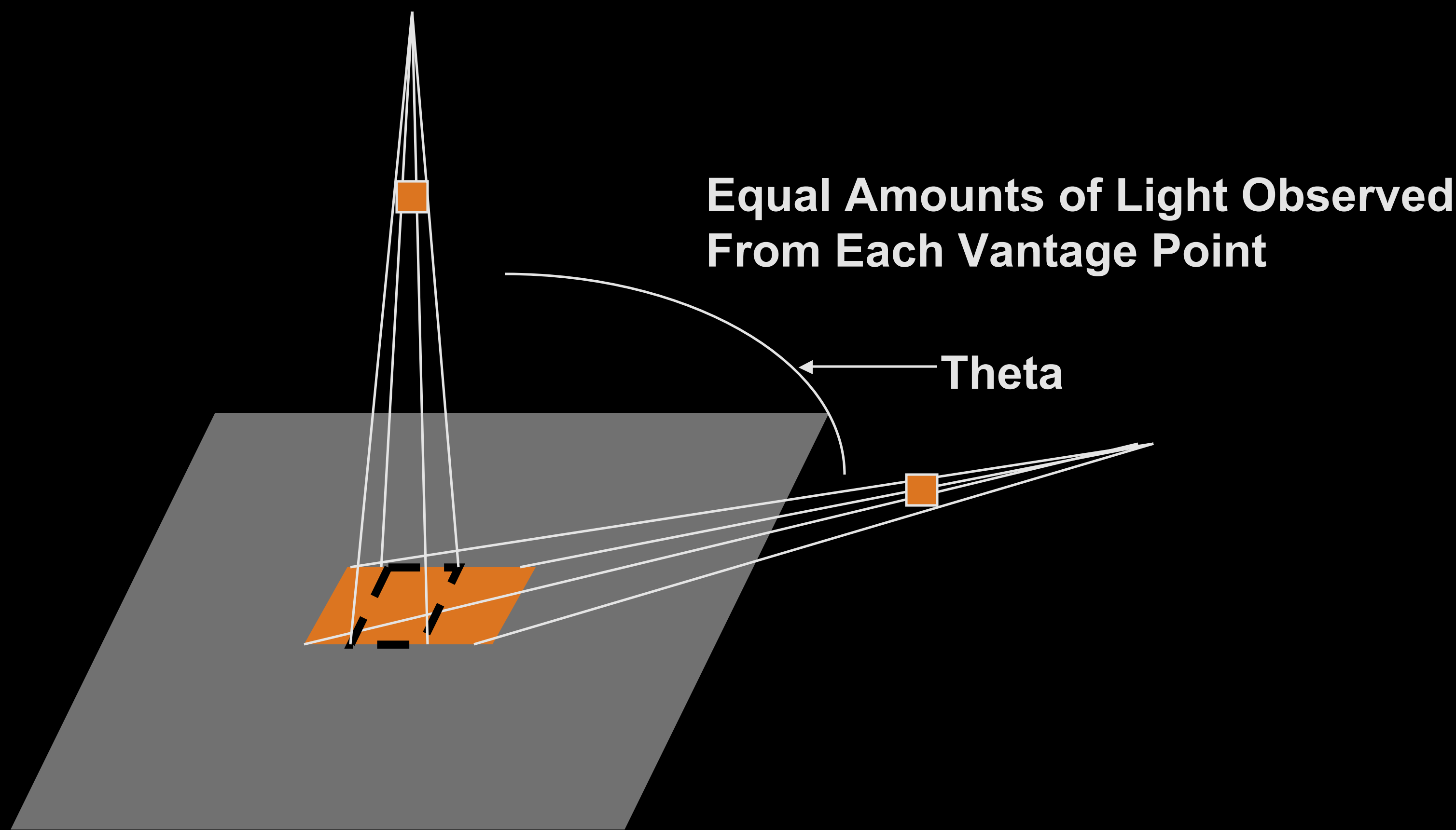
Pseudo-Radiance

- Consider two definitions:
  - Radiance:
    power per unit foreshortened area emitted into a solid angle
  - Pseudo-radiance
    power per unit area emitted into a solid angle
  - Why should we work with radiance rather than pseudo-radiance?
    - Only reason: Radiance is more closely related to our intuitive notion of “brightness”.

Lambertian Surfaces

- A particular point P on a Lambertian (perfectly matte) surface appears to have the same brightness no matter what angle it is viewed from.
  - Piece of paper
  - Matte paint
- Doesn’t depend upon incident light angle.
- What does this say about how they emit light?

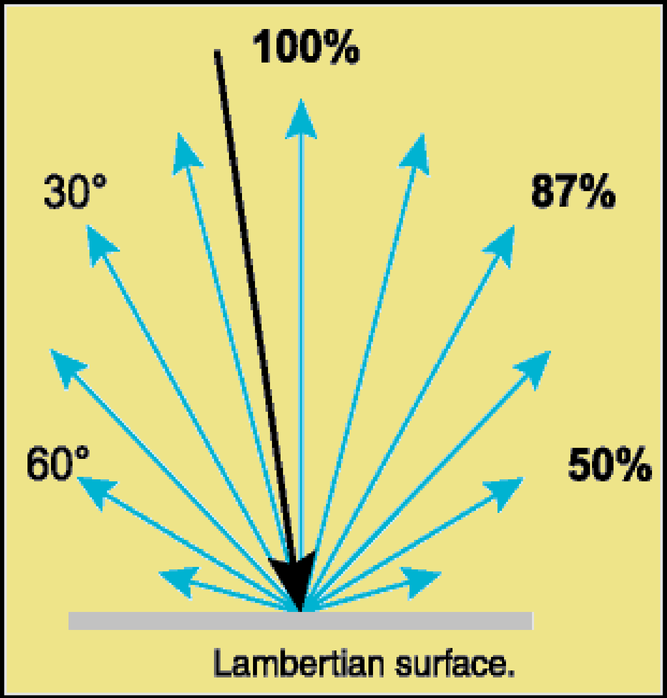
Lambertian Surfaces



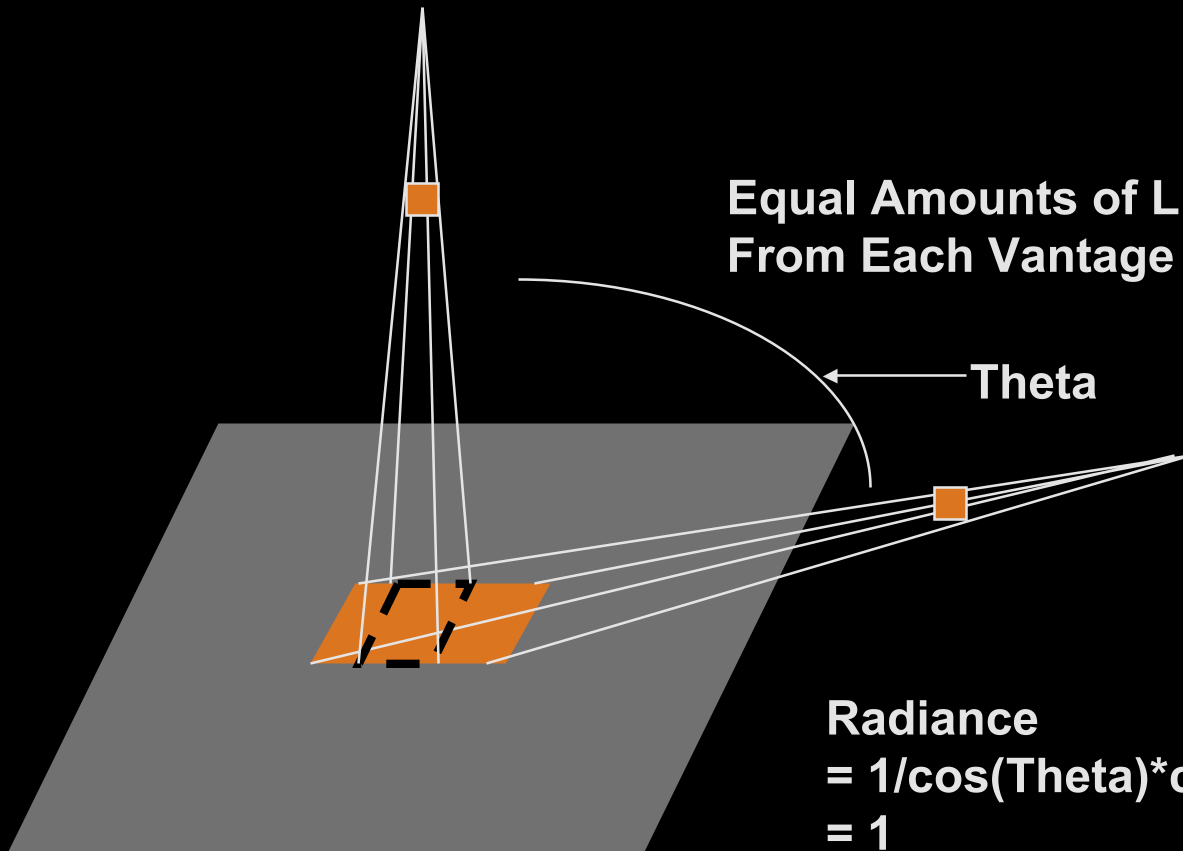
Area of black box = 1
Area of orange box = 1/cos(Theta)
Foreshortening rule.

Lambertian Surfaces

Relative magnitude of light scattered in each direction.
Proportional to cos (Theta).



Lambertian Surfaces



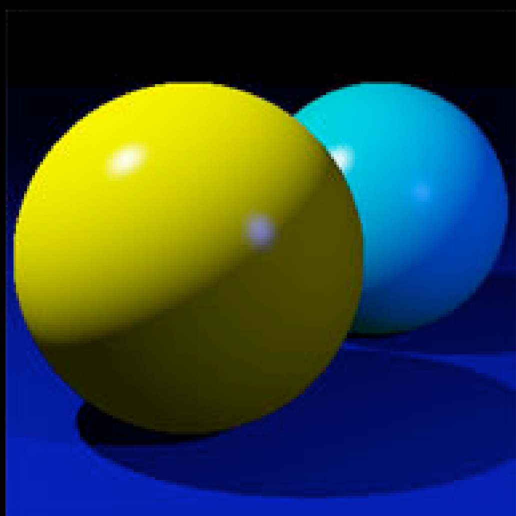
Equal Amounts of Light Observed From Each Vantage Point

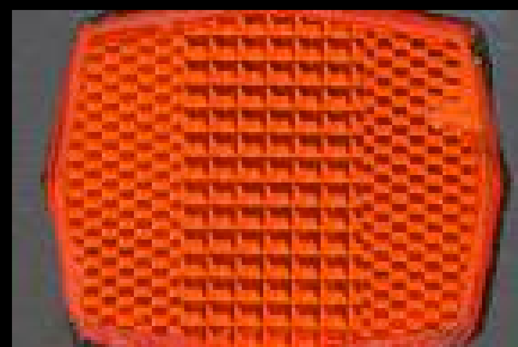
Radiance
= 1/cos(Theta)*cos(Theta)
= 1

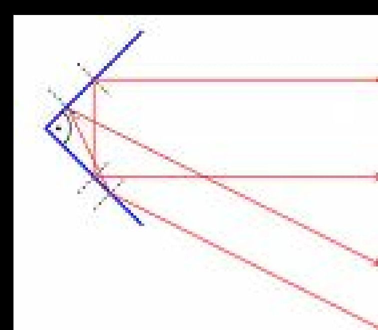
Area of black box = 1
Area of orange box = 1/cos(Theta)
Foreshortening rule.

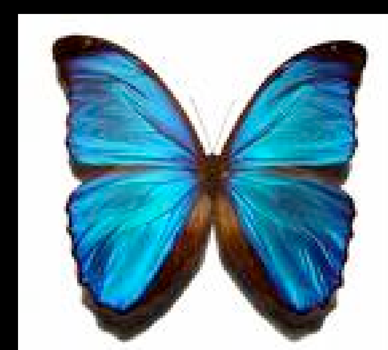
The BRDF

The bidirectional reflectance distribution function.







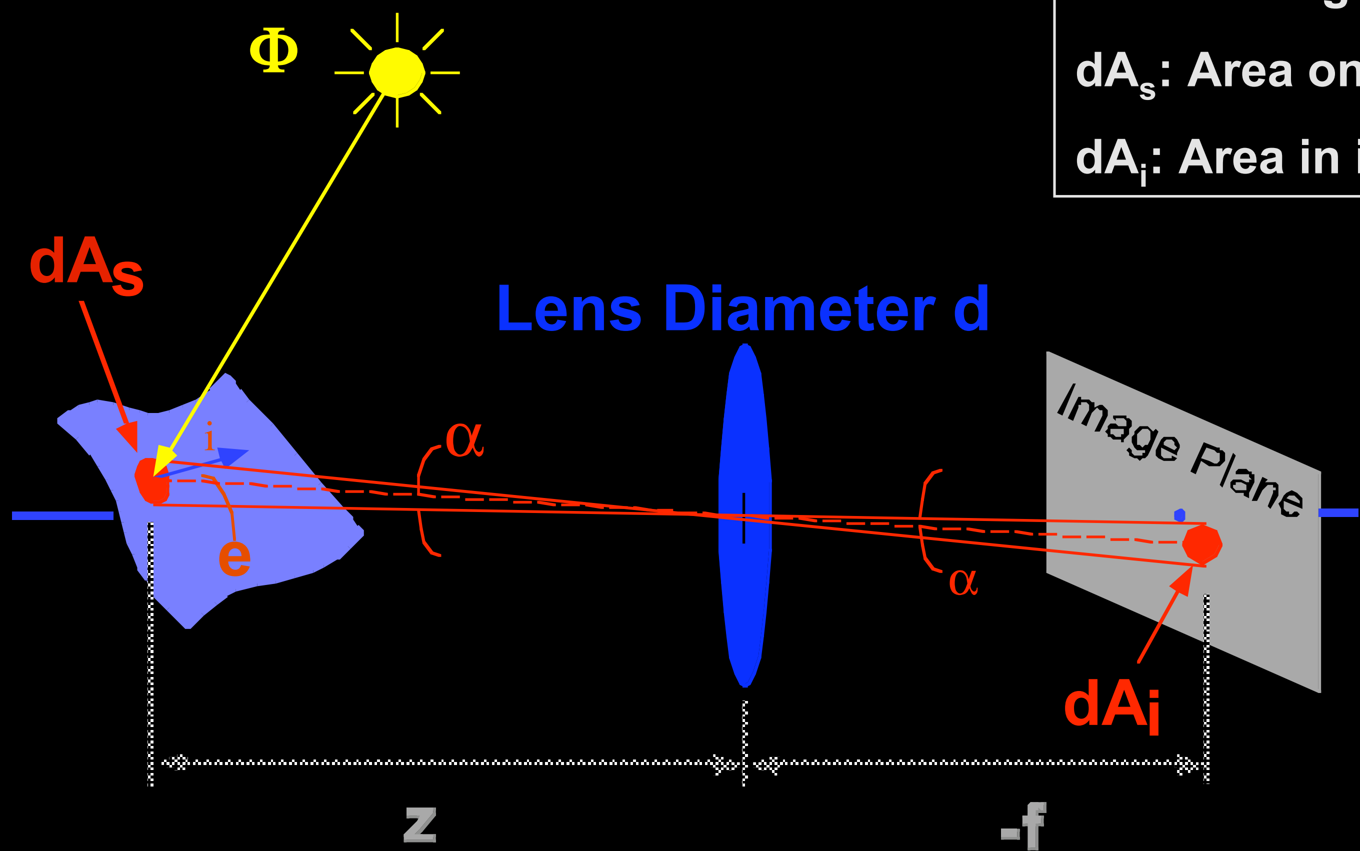


- Goal: Relate the radiance of a surface to the irradiance in the image plane of a simple optical system.

α: Solid angle of patch

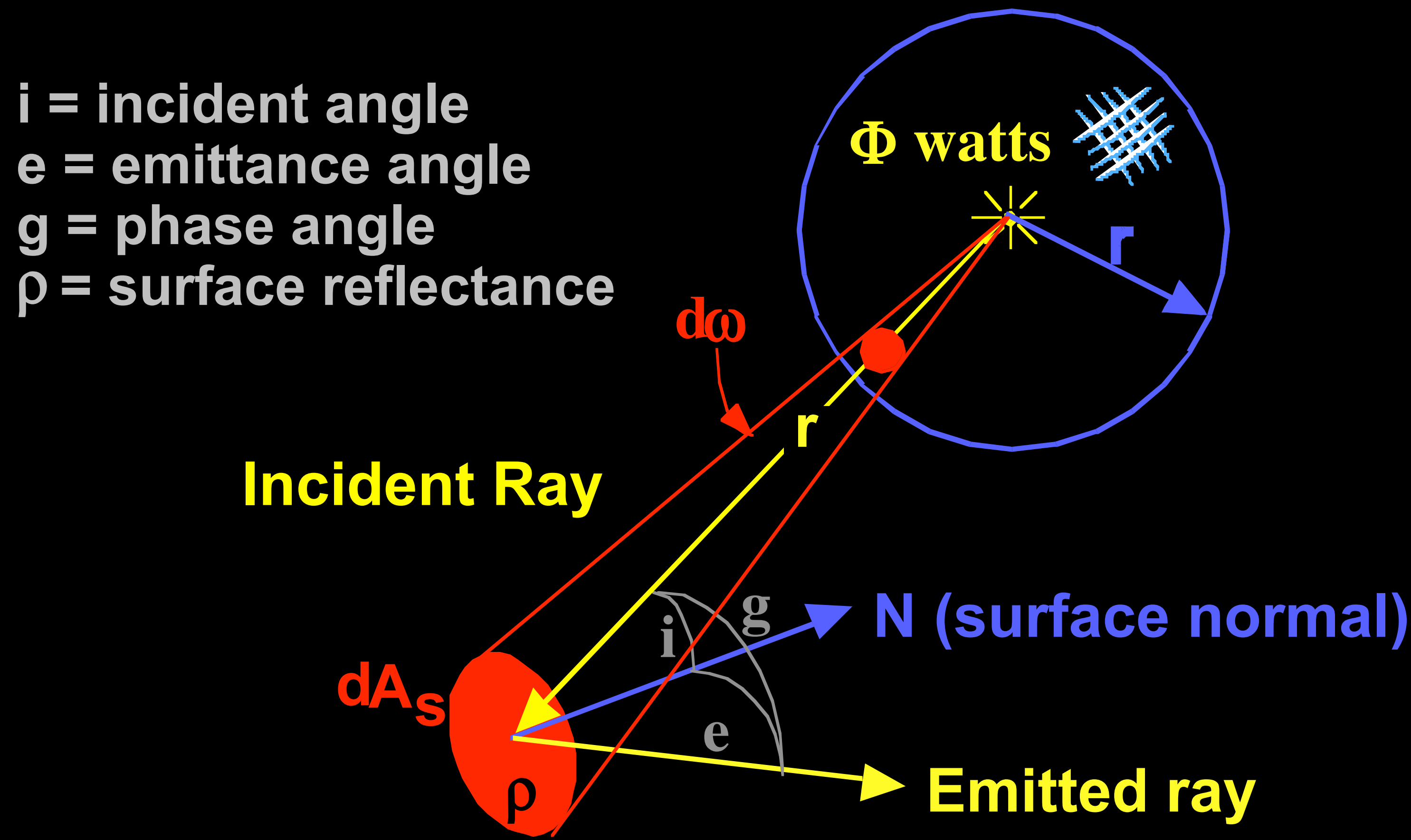
dA_s: Area on surface

dA_i: Area in image



Light at the Surface

- E = flux incident on the surface (irradiance) = $\frac{d\Phi}{dA}$



- We need to determine dΦ and dA

Reflections from a Surface I

dA

- $dA = dA_s \cos i$ {foreshortening effect in direction of light source}

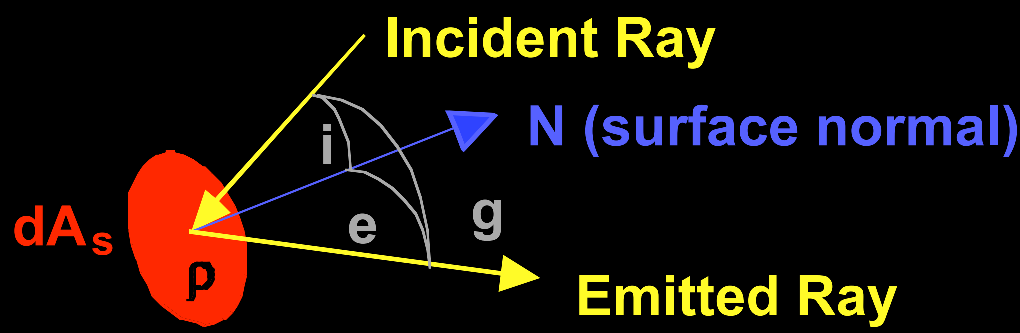
$d\Phi$

- $d\Phi$ = flux intercepted by surface over area dA
  - dA subtends solid angle $d\omega = dA_s \cos i / r^2$
  - $d\Phi = R\, d\omega = R\, dA_s \cos i / r^2$
  - $E = d\Phi / dA_s$

Surface Irradiance: $E = R \cos i / r^2$

Reflections from a Surface II

- Now treat small surface area as an emitter
  - ….because it is bouncing light into the world
- How much light gets reflected?

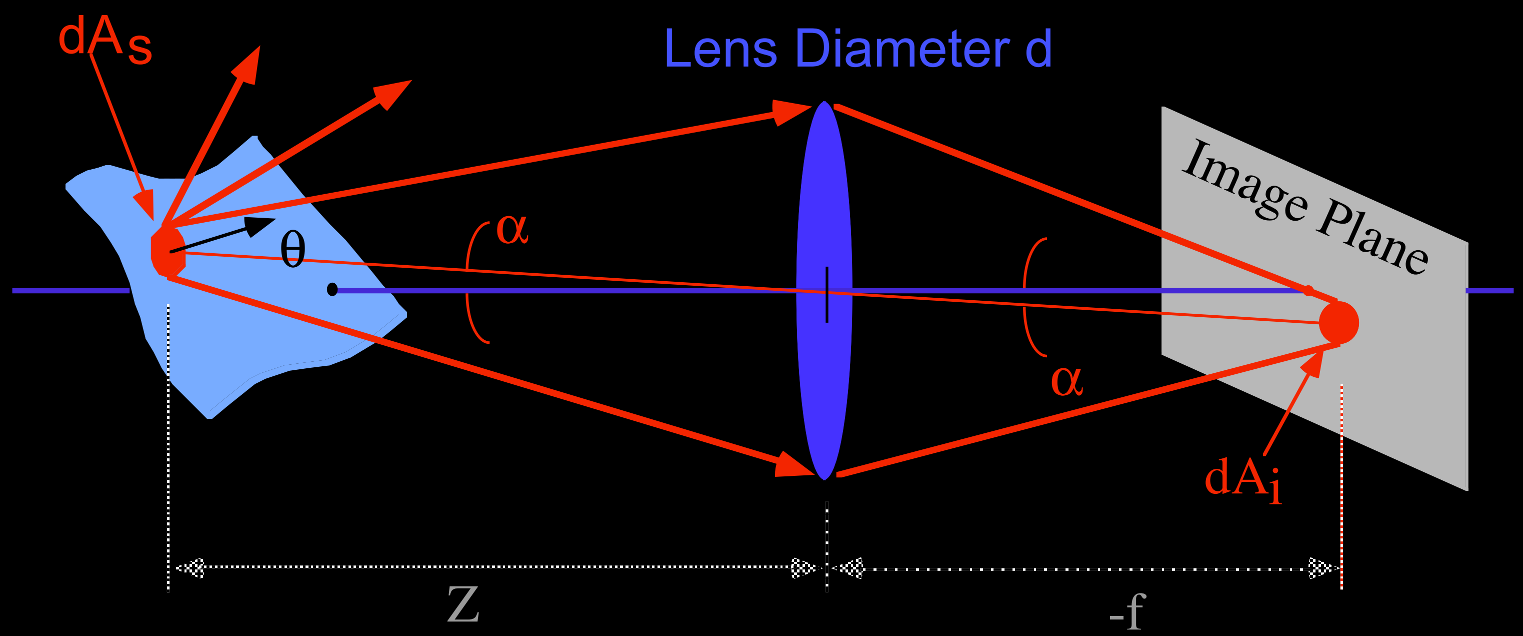


- E is the surface irradiance
- L is the surface radiance = luminance
- They are related through the surface reflectance function:

$$\frac{L_s}{E} = \rho(i,e,g,\lambda)$$

May also be a function of the wavelength of the light

Power Concentrated in Lens



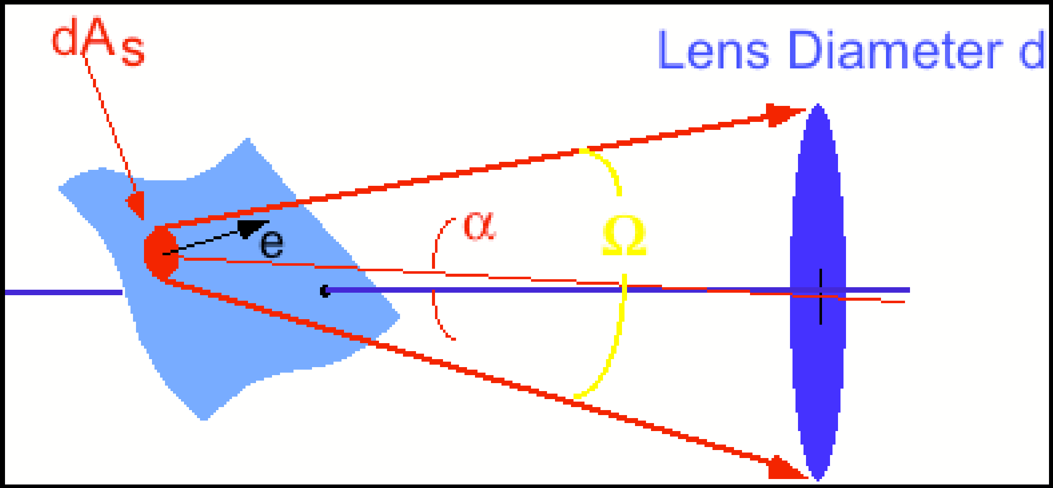
$$L_s = \frac{d^2\Phi}{dA_s d\omega}$$

Luminance of patch (known from previous step)

What is the power of the surface patch as a source in the direction of the lens?

$$d^2\Phi = L_s \, dA_s \, d\omega$$

Through a Lens Darkly



- In general:
  - L_s is a function of the angles i and e.
  - Lens can be quite large
  - Hence, must integrate over the lens solid angle to get $d\Phi$

$$d\Phi = dA_s \int_{\Omega} L_s \, d\Omega$$

Simplifying Assumption

- Lens diameter is small relative to distance from patch

$$d\Phi = dA_s \int_{\Omega} L_s \, d\Omega$$

L_s is a constant and can be removed from the integral

$$d\Phi = dA_s L_s \int_{\Omega} d\Omega$$

Surface area of patch in direction of lens

$$= dA_s \cos e$$

Solid angle subtended by lens in direction of patch

$$= \frac{\text{Area of lens as seen from patch}}{(\text{Distance from lens to patch})^2}$$

$$= \frac{\pi (d/2)^2 \cos \alpha}{(z / \cos \alpha)^2}$$

Putting it Together

$$d\Phi = dA_s \int_{\Omega} L_s \, d\Omega$$

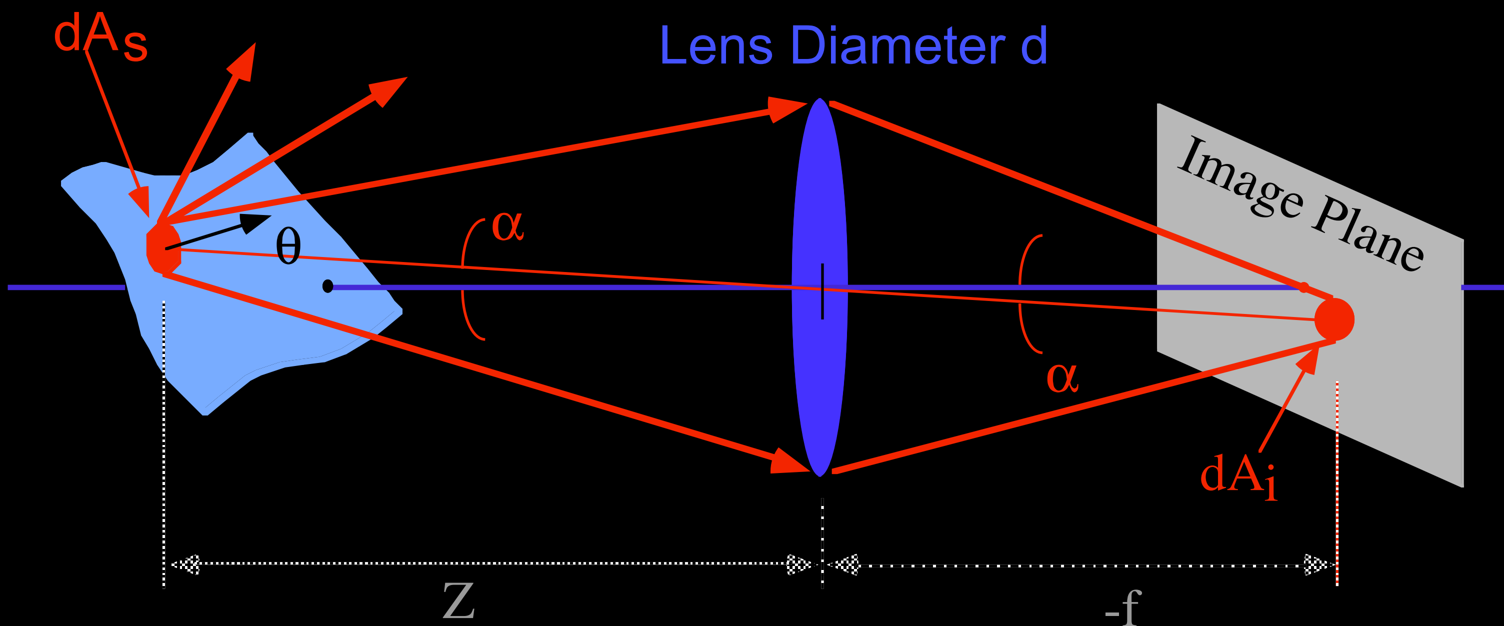
$$= dA_s \cos e \; L_s \frac{\pi (d/2)^2 \cos\alpha}{(z / \cos\alpha)^2}$$

- Power concentrated in lens:

$$d\Phi = \frac{\pi}{4} L_s \, dA_s \left[\frac{d}{z}\right]^2 \cos e \, \cos^3\alpha$$

- Assuming a lossless lens, this is also the power radiated by the lens as a source.

Through a Lens Darkly

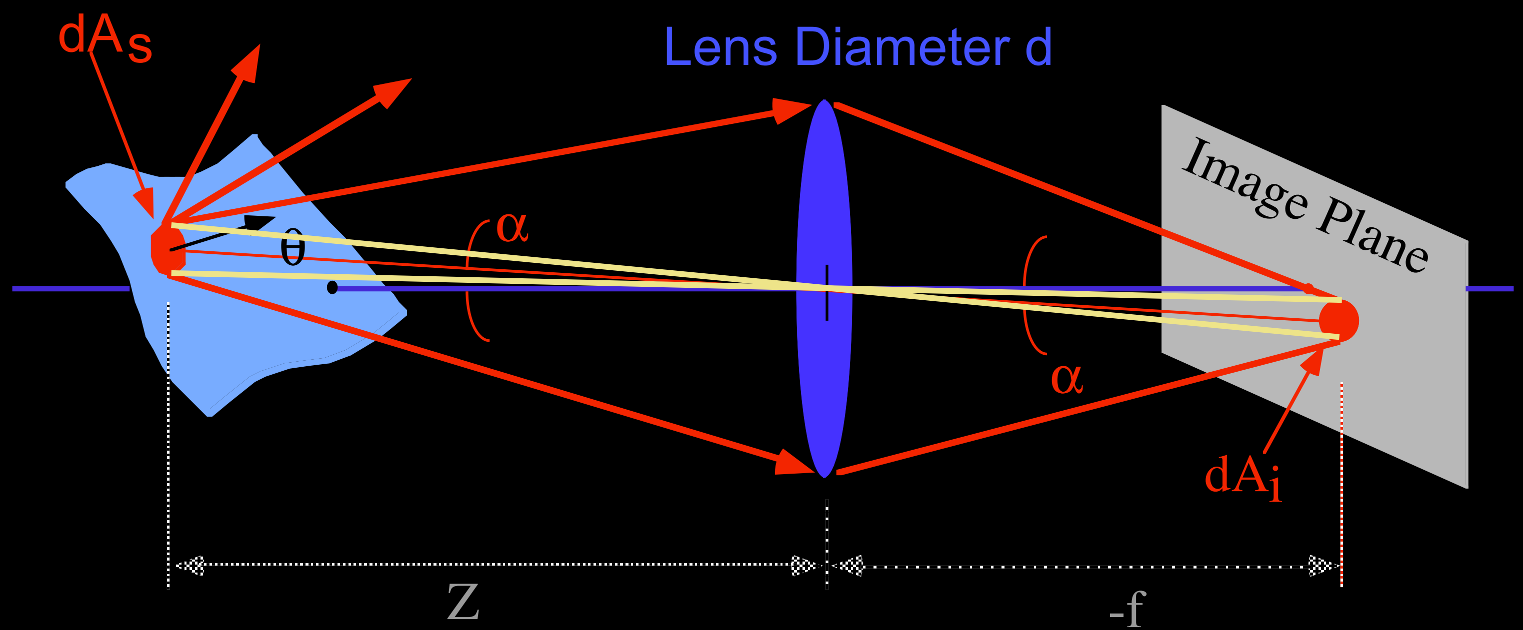


- Image irradiance at $dA_i = \dfrac{d\Phi}{dA_i} = E_i$

$$E_i = L_s \frac{dA_s}{dA_i} \frac{\pi}{4} \left[\frac{d}{Z} \right]^2 \cos e \cos^3 \alpha$$

ratio of areas

Patch ratio



The two solid angles are equal

$$\frac{dA_s \cos e}{(Z / \cos\alpha)^2} = \frac{dA_i \cos\alpha}{(-f / \cos\alpha)^2} \Rightarrow \frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos e}\left(\frac{Z}{-f}\right)^2$$

The Fundamental Result

- Source Radiance to Image Sensor Irradiance:

$$\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos e}\left(\frac{Z}{-f}\right)^2$$

$$E_i = L_s \frac{dA_s}{dA_i} \frac{\pi}{4}\left(\frac{d}{Z}\right)^2 \cos e \cos^3\alpha$$

$$E_i = L_s \frac{\cos\alpha}{\cos e}\left(\frac{Z}{-f}\right)^2 \frac{\pi}{4}\left(\frac{d}{Z}\right)^2 \cos e \cos^3\alpha$$

$$E_i = L_s \frac{\pi}{4}\left(\frac{d}{-f}\right)^2 \cos^4\alpha$$

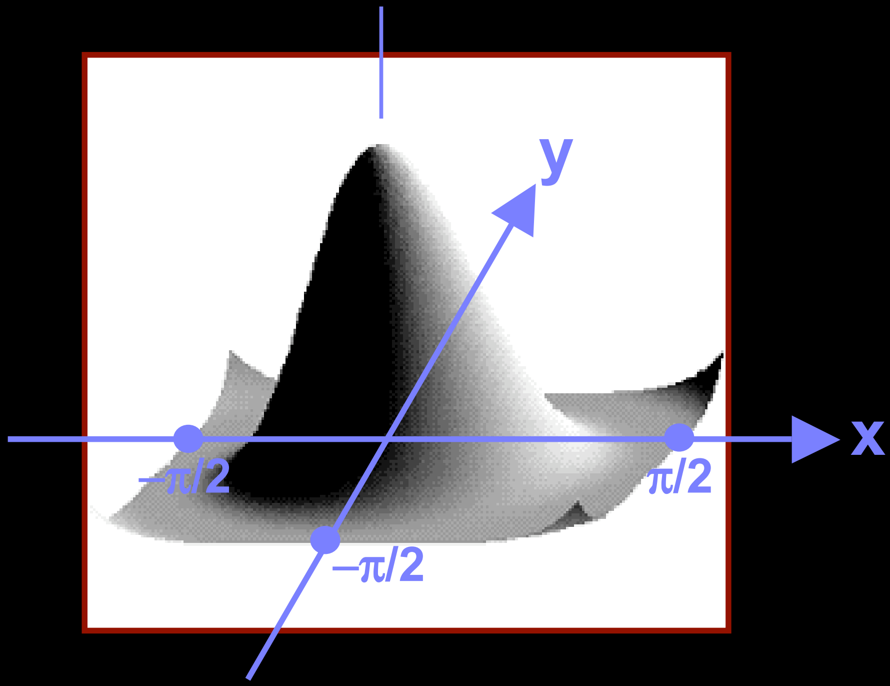
Radiometry Final Result

$$E_i = L_s \frac{\pi}{4}\left(\frac{d}{-f}\right)^2 \cos^4\alpha$$

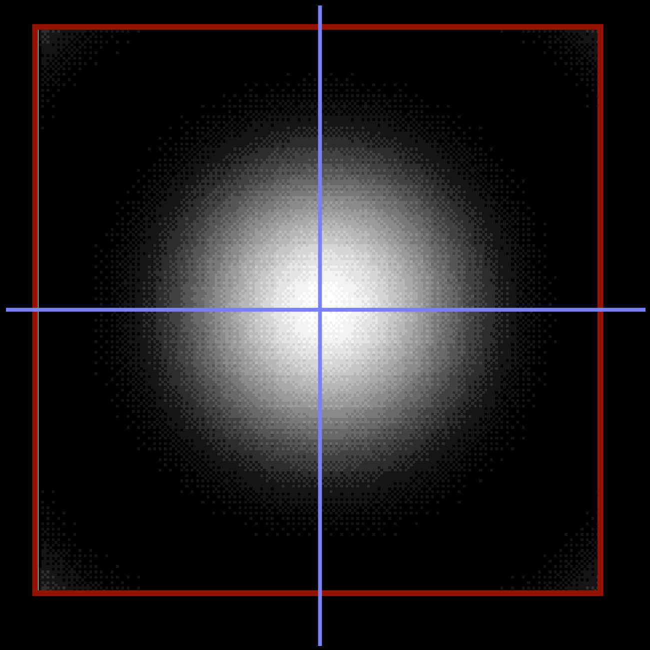
- Image irradiance is a function of:
  - Scene radiance L_s
  - Focal length of lens f
  - Diameter of lens d
    - f/d is often called the 'effective focal length' of the lens
  - Off-axis angle α

$Cos^4\alpha$ Light Falloff

Lens Center

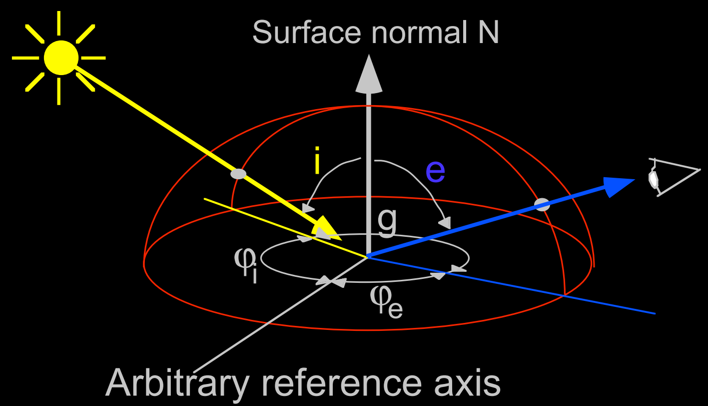


Top view shaded by height



Limitation of Radiometry Model

- Surface reflection ρ can be a function of viewing and/or illumination angle

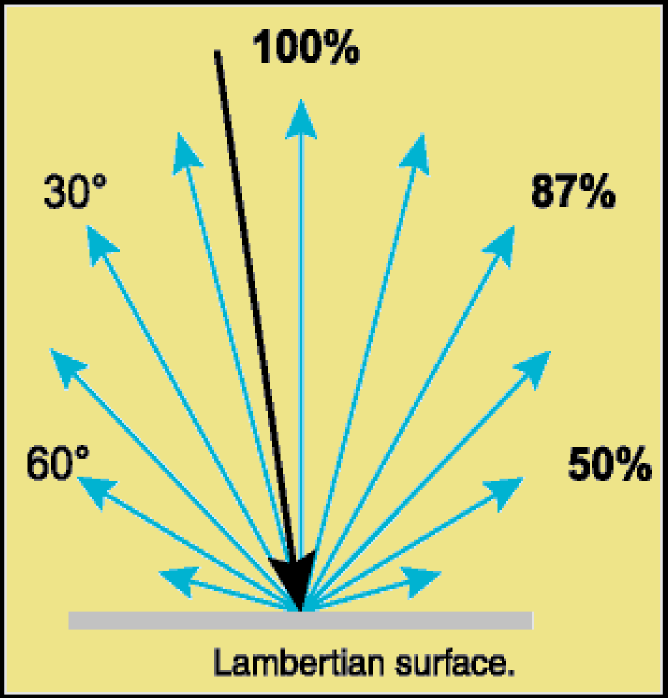


$$\rho(i,e,g,\varphi_e,\varphi_i) = \frac{dL(e,\varphi_e)}{dE(i,\varphi_i)}$$

- ρ may also be a function of the wavelength of the light source
- Assumed a point source (sky, for example, is not)

Lambertian Surfaces

- The BRDF for a Lambertian surface is a constant
  - $\rho(i,e,g,\varphi_e,\varphi_i) = k$
  - function of cos e due to the foreshortening effect
  - k is the 'albedo' of the surface
  - Good model for diffuse surfaces
- Other models combine diffuse and specular components (Phong, Torrance-Sparrow, Oren-Nayar)



BRDF

- Ron Dror's thesis

Reflection parameters

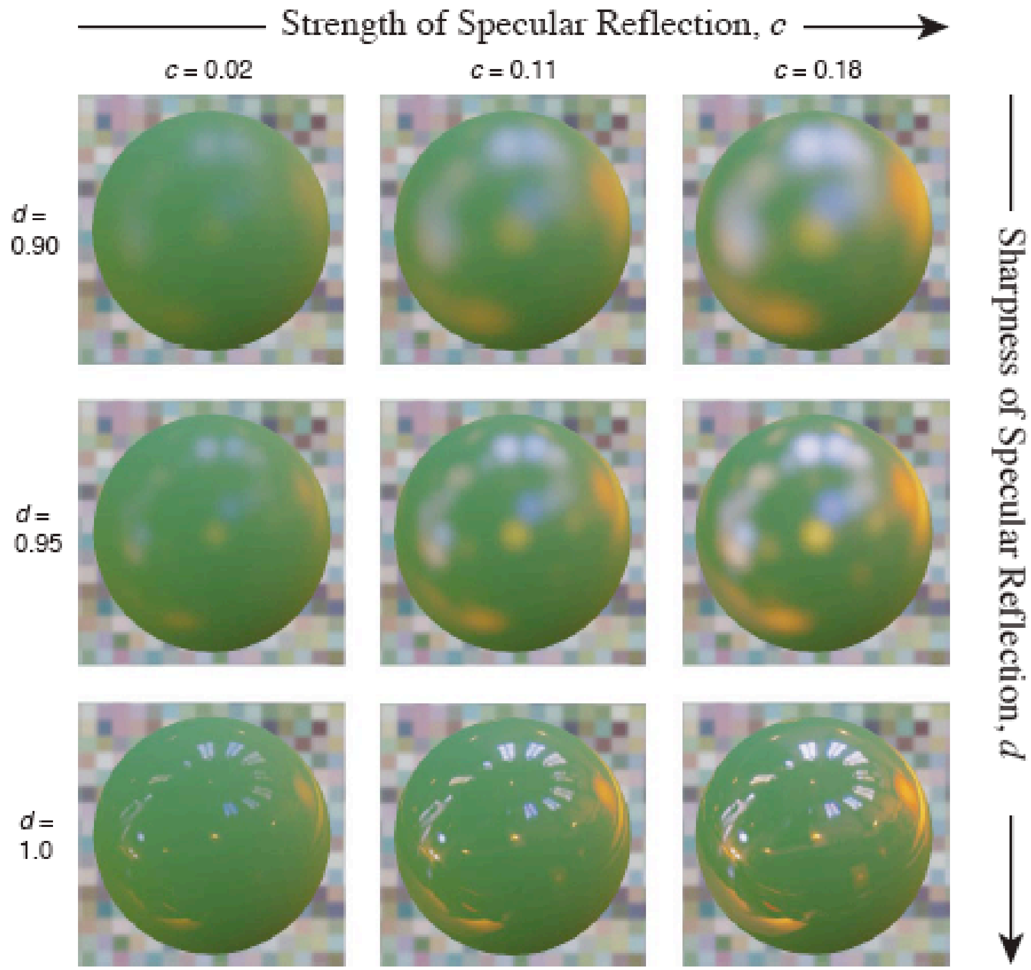
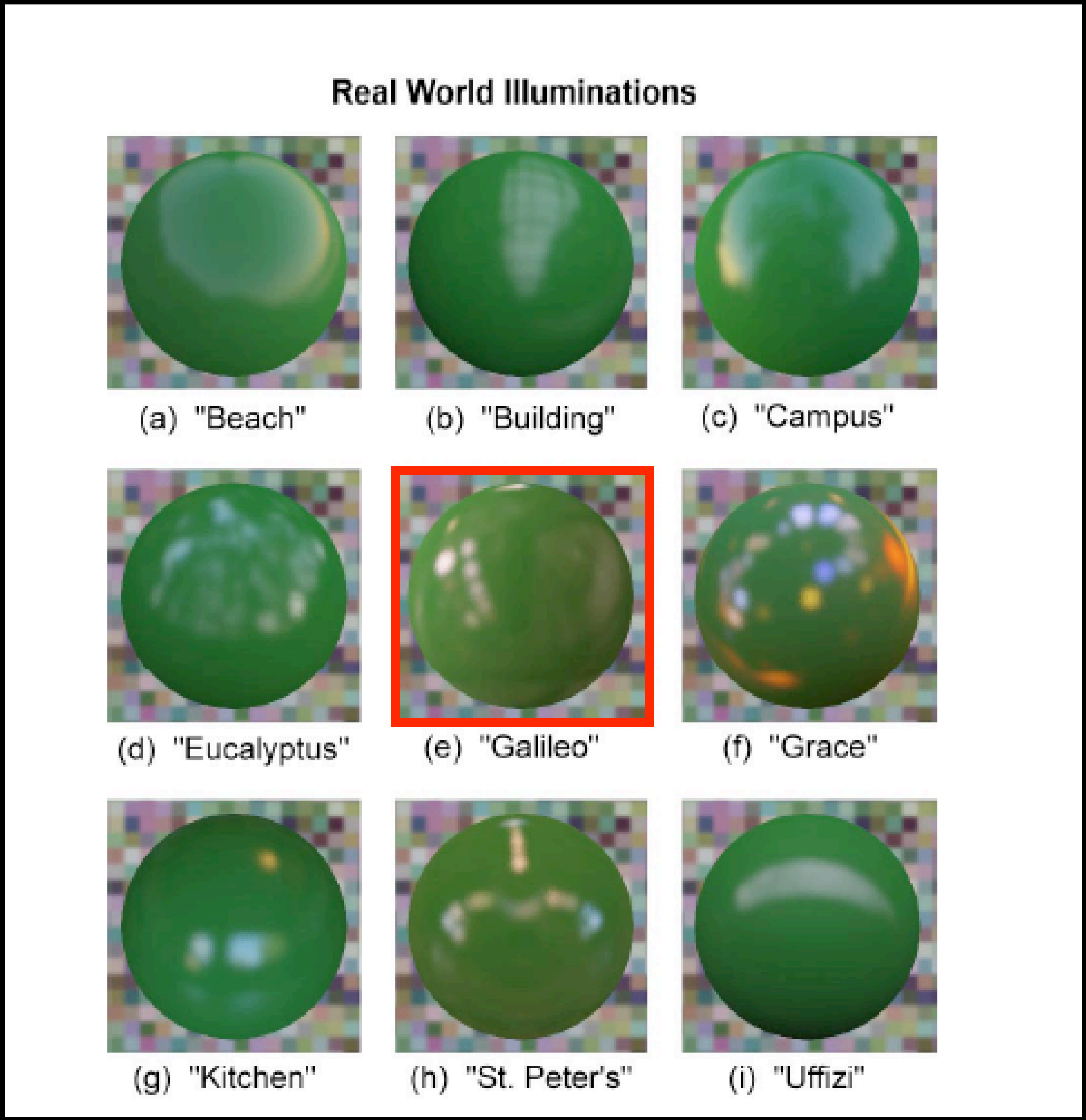
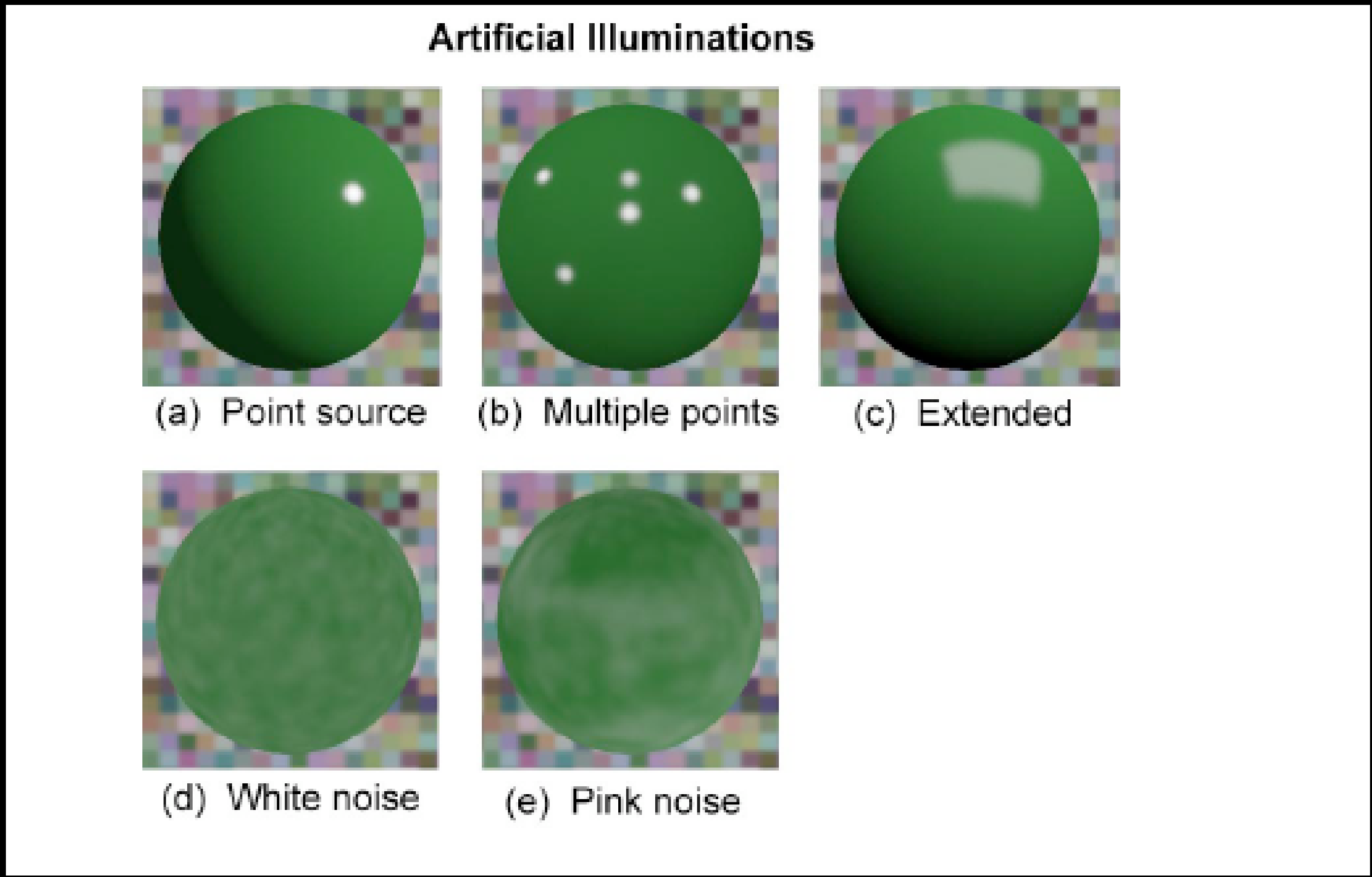


Figure 2.6. Grid showing range of reflectance properties used in the experiments for a particular real-world illumination map. All the spheres shown have an identical diffuse component. In Pellacini's reparameterization of the Ward model, the specular component depends on the c and d parameters. The strength of specular reflection, c, increases with ρ_s, while the sharpness of specular reflection, d, decreases with α. The images were rendered in *Radiance*, using the techniques described in Appendix B.

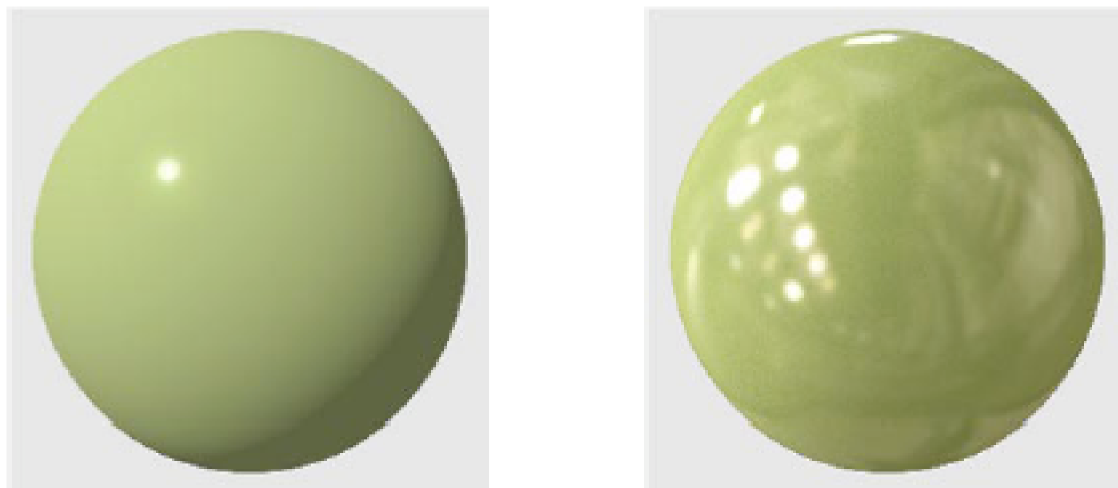
Real World Light Variation



Fake Light and Real Light



Simple and Complex Light



(a) (b)

Figure 2.9. (a) A shiny sphere rendered under illumination by a point light source. (b) The same sphere rendered under photographically-acquired real-world illumination. Humans perceive reflectance properties more accurately in (b).

BRDF again

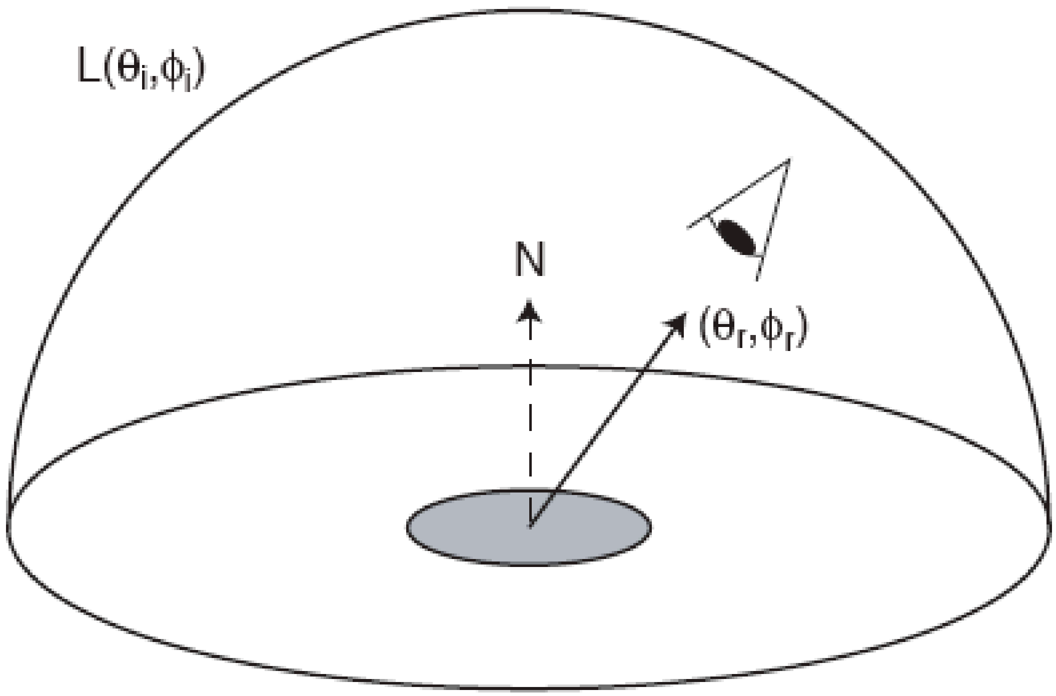


Figure 3.1. A viewer observes a surface patch with normal N from direction (θ_r, ϕ_r). $L(\theta_i, \phi_i)$ represents radiance of illumination from direction (θ_i, ϕ_i). The coordinate system is such that N points in direction $(0, 0)$.

$$B(\theta_r, \phi_r) = \int_{\phi_i=0}^{2\pi} \int_{\theta_i=0}^{\pi/2} L(\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos\theta_i \sin\theta_i \, d\theta_i \, d\phi_i,$$

Figure 3.6. A photograph of a matte sphere, shown against a uniform gray background. This image could also be produced by a chrome sphere under appropriate illumination, but that scenario is highly unlikely.